STARTING WITH

CONFIDENCE

A-LEVEL MATHS AT BUDMOUTH

Name:...

This booklet has been designed to help you to bridge the gap between GCSE Maths and AS Maths.

Good mathematics is not about how many answers you know…Its how you behave when you don't know.

You need to <u>complete</u> this booklet before your course commences in September. ***You will be tested on these topics to ensure you have the skills needed to be successful at A-Level Maths .***

Please make sure your answers to the <u>mock test</u> found on page 28, are handed in to your Maths teacher on the first lesson. This work will be marked and feedback will be given.

HOW IS A-LEVEL MATHS DIFFERENT FROM GCSE

GCSE	A Level
• You use square paper. • It's the answer that matters most, but you should show working. • Fractions and decimals are equally nice and mixed numbers are fine. • If you're good at Maths, you can do well without trying.	• You use lined paper. • It's the method that matters most, not the answer. Often, you are given the answer and need to explain the method. • Fractions are MUCH better than decimals and mixed numbers are *not* nice! • You will do a <u>lot</u> of study outside of class and concepts will take longer to understand.

WHEN (NOT IF) YOU GET STUCK…..

Studying Maths at Advanced Level is about learning how to solve problems. The first stage of solving a problem is being stuck, so you should expect to get stuck whilst working through this booklet. Some of these topics may seem unfamiliar to you but they are all GCSE level topics and you need to be able to do all these techniques before you start AS Maths.

So, when you get stuck…

- **Look again at the examples. Maybe there is one which shows you how to solve your problem?**
- **Have you made a mistake? It might be that your method is correct but you've made an error in your working somewhere.**
- **Try looking up the topic in a GCSE higher tier textbook or revision guide (you can get these from your local library)**
- **Post a question on a forum (*www.thestudentroom.co.uk*)**
- **Try some of the following websites which are excellent for revision:**
    - **www.hegartymaths.com**
    - **www.examsolutions.net**
    - **www.mymaths.co.uk (U: *budmouth* P: *square*)**

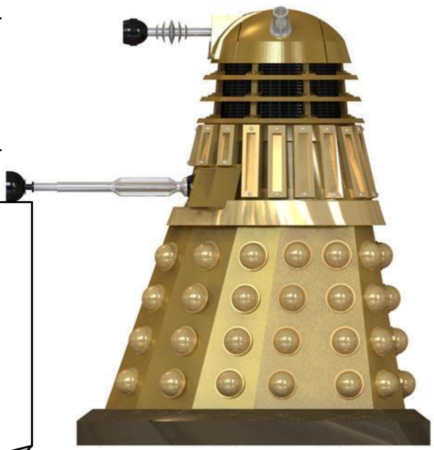
CONTENTS

PART A – LEARNING TO AVOID COMMON ALGEBRAIC 'MISTAKES'.

We all make occasional mistakes when manipulating algebra and learning to make fewer mistakes (and finding the ones you have made!) is an important part of the study of maths at A Level. However, there are also mistakes that aren't mistakes at all but are actually the result of a deeply held misunderstanding about the laws of algebra. These misunderstandings need to be exterminated as soon as possible. Do you understand why these examples are wrong?

WRONG!! Exterminate!!

EXTERMINATE!!
Exterminate!!

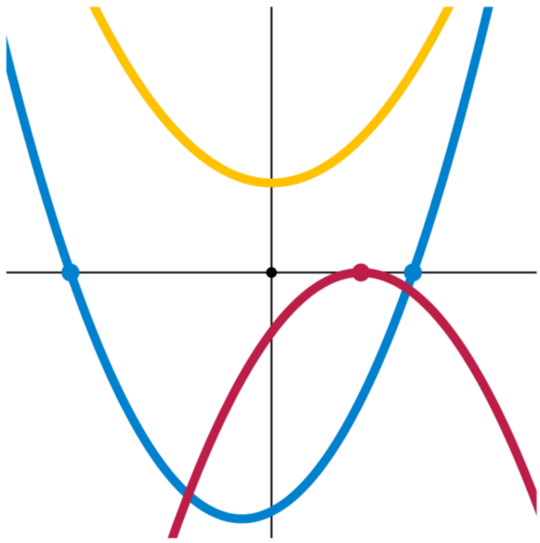


WRONG!! Exterminate!!

WRONG!! Exterminate!!

PART B – DEVELOPING CONFIDENCE WITH QUADRATICS

A quadratic is any algebraic expression with some x^2 bits and some x bits and a number i.e. $ax^2 + bx + c$. In your study of GCSE maths you will have met, and learned to solve, quadratic equations. In order to cope with the demands of AS Maths, you need to be <u>confident</u> when working with quadratics and this is something we have found that causes a lot of problems in the transition from GCSE to AS maths. This part of the booklet will outline everything you need to remember about quadratics and give you a chance to practise building your confidence with these important equations.



You should recognise these curves as quadratic curves. On page 17 of this booklet you will learn what the discriminant is and how to measure and interpret it for any quadratic. ☺

- You should be able to complete this entire booklet WITHOUT using a calculator. You won't be allowed one in your first exam.

Suggested Study Plan

Do you feel really confident with all of the A and A* techniques that you learnt at GCSE?

No

We will not have time to cover these techniques in class next year, but you ARE required to know them when you start AS. Therefore you need to practise over the summer. The exercises in this booklet are designed to help you do that. It would be better if you practise little and often, rather than a lot all at once.

Yes

Do the mini-tests at the end of Part A (pg 15) and Part B (pg 26). Did you score …?

Less than 60%

You need to be much more confident with these techniques before September. Work through the whole booklet carefully (again) and use the "When You Get Stuck" tips on page 2 to help you make progress.

60-90%

Pretty good but there are obviously some areas you still need to work on. Identify these sections in the booklet. Go through the examples carefully and do the exercises (again!)

More than 90%

This is a really good score – **well done!** Go over your mistakes. What mistakes did you make? How could you avoid making them in the future? Use the examples and exercises in the booklet to help you.

Finally, make sure you have gone through the booklet and collected together the common mistakes (indicated by daleks).

Work through the exercises in Part A until you are confident with all of the techniques.

On a different day, do the section A mini-test. Did you pass (or do better than last time)?

No

Yes

Work through the exercises in Part B until you are confident with all of the techniques.

On a different day, do the section B mini-test. Did you pass (or do better than last time)?

No

Yes

In the last week of the holidays, do the "Are you Ready for AS Test?" (pg 27). Did you score…

Less than 60%

Go through the exercises again where you are having problems. Discus with with a teacher on registration day whether A-Level

60-80%

Identify the areas where you are making mistakes. Go through the relevant exercises again. You must turn up to the after-school maths revision class.

More than 80%

Well done – you have the necessary building blocks in place in order to start AS Maths with confidence.

PART A – SECTION 1 : FRACTIONS

TOP TIP! Never use a slanted line like this because the x will try to escape by moving right a bit and growing .
It is much harder for the x to escape if you use a horizontal line.

TOP TIP! You will make fewer mistakes if you write things next to each other like $3x$ rather than and rather than .

TOP TIP! If you want to multiply a fraction by a number, you can write the number as a fraction by putting it over 1:   . This avoids the possibility of making the **common mistake** that

Exercise 1 In the spaces available, carry out the following, leaving your answer as a single fraction.

(1) $\frac{3x}{4}\times 5$	(2) $\frac{2}{x}+\frac{3}{x^2}$ (hint: make the denominators the same by multiplying top and bottom of $\frac{2}{x}$ by x, then add the numerators)	(3) $\frac{3x}{2}\div 5$
Answers at the back		

Tick when you’re correct…..	Tick when correct………….	Tick when correct………….

PART A – SECTION 2 : INDICES

WRONG!!!
Students often think that if there is multiplication in the powers it must correspond to multiplication.
In fact, or .

WRONG!!!
Students often think that if there is addition in the power it must correspond to addition. In fact, .

Exercise 2

Evaluate the following, tick the boxes when they are correct:

THE RULES OF INDICES

Rules: $\boldsymbol{a^m a^n = a^{m+n}}$ $\quad \dfrac{a^m}{a^n} = a^{m-n}$ $\quad \boldsymbol{(a^m)^n = a^{mn}}$

Also: $\boldsymbol{(ab)^n = a^n b^n}$ $\quad \boldsymbol{a^0 = 1}$ $\quad \boldsymbol{a^1 = a}$

A <u>negative</u> power indicates a <u>reciprocal</u> e.g 6^{-2} means $\dfrac{1}{6^2} = \dfrac{1}{36}$ and 5^{-3} means $\dfrac{1}{5^3} = \dfrac{1}{125}$	(1) 2^{-6}	☐
	(2) $9^{-\frac{1}{2}}$	☐
<u>Example</u>: $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 2$ ☐ Tick the box when you understand!	(3) $81^{-\frac{1}{4}}$	☐
A <u>fractional</u> power indicates a <u>root</u>	(4) $4^{\frac{5}{2}}$	☐

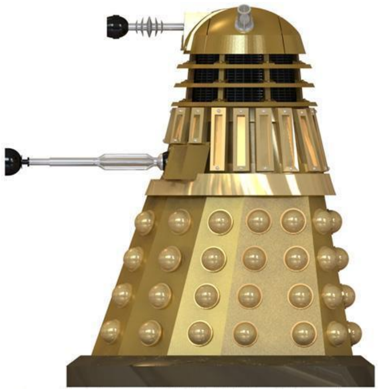
a power of $\frac{1}{2}$ means 'square root'. $25^{\frac{1}{2}} = \sqrt{25} = 5$ a power of $\frac{1}{3}$ means 'cube root'. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$	(5) $32^{\frac{3}{5}}$	
<u>Example</u>: $144^{-\frac{1}{2}} = \frac{1}{144^{\frac{1}{2}}} = \frac{1}{\sqrt{144}} = \frac{1}{12}$ Tick the box when you understand.	(6) $16^{-\frac{7}{4}}$	

INDICES CONTINUED (WHAT YOU NEED FOR AS LEVEL)

It is very useful to mathematicians to be able to write algebraic expressions in different ways and one of the most important ways is in the form (number) x^{power}

WRONG!! Actually, it's

WRONG!! Actually, it's



Examples of writing things in the form αx^n. Tick the box when you understand.	**Now try Exercise 3:** Write these in the form αx^n. Tick when correct.
$\frac{2x}{3} = \left(\frac{2}{3}\right)\left(\frac{x}{1}\right)$ $= \frac{2}{3}x$	(1) $\frac{x}{5} =$
$\frac{2}{5x} = \left(\frac{2}{5}\right)\left(\frac{1}{x}\right)$ $= \frac{2}{5}x^{-1}$	(2) $\frac{3}{2\sqrt{x}} =$
$\frac{x}{3\sqrt{x}} = \left(\frac{1}{3}\right)\left(\frac{x}{\sqrt{x}}\right)$ $= \frac{1}{3}x^{1-\frac{1}{2}}$ $= \frac{1}{3}x^{\frac{1}{2}}$	(3) $\frac{\sqrt{x}}{3x^2} =$

$2\sqrt{16x^3} = 2\sqrt{16}\sqrt{x^3}$ $= 8\left(x^3\right)^{\frac{1}{2}}$ $= 8x^{\frac{3}{2}}$	(4) $\sqrt[3]{8x^2} =$
$\dfrac{2+x}{\sqrt{x}} = \dfrac{2}{\sqrt{x}} + \dfrac{x}{\sqrt{x}}$ $= \left(\dfrac{2}{1}\right)\left(\dfrac{1}{x^{\frac{1}{2}}}\right) + x^{\left(1-\frac{1}{2}\right)}$ $= 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}$	(5) $\dfrac{2\sqrt{x}+4}{x^2} =$

Note: This one has two terms so is written in the form

More practise of **the most important type of indices…** Write these in the form $\alpha x^n + \beta x^m$. Tick the boxes when they are correct.		
(6) $\frac{2x-4}{3x^2} = \frac{2x}{3x^2} - \frac{4}{3x^2}$ $= \left(\frac{2}{3}\right)\left(\frac{x}{x^2}\right) - \left(\frac{4}{3}\right)\left(\frac{1}{x^2}\right)$ $= \frac{2}{3}x^{-1} - \frac{4}{3}x^{-2}$ UNDERSTAND?............. ☐	(7) $\frac{1-4x}{4x^3} =$ ☐	(8) $\frac{1-4\sqrt{x}}{x} =$ ☐
(9) $\frac{x^2-3}{\sqrt{x}} =$ ☐	(10) $\frac{x-2}{x^2} =$ ☐	(11) $\frac{2+\sqrt{x}}{\sqrt{x}} =$ ☐
(12) $\frac{2x+4}{4x} =$ ☐	(13) $\frac{\sqrt{x}+6}{3x^2} =$ ☐	(14) $\frac{2x-1}{x^2} =$ ☐

Examples of solving index equations by doing the same thing to both sides. Tick when understood.	**Exercise 3 continued:** Solve each of the following equations for *x*. Tick when correct.
$x^{-\frac{1}{2}} = 3$ $\left(x^{-\frac{1}{2}}\right)^{-1} = 3^{-1}$ $x^{\frac{1}{2}} = \frac{1}{3}$ $\left(x^{\frac{1}{2}}\right)^{2} = \left(\frac{1}{3}\right)^{2}$ $x = \frac{1^2}{3^3} = \frac{1}{9}$	(15) $x^{-\frac{2}{3}} = 9$
$x^{\frac{2}{5}} = 2$ $\left(x^{\frac{2}{5}}\right)^{5} = 2^{5}$ $x^2 = 32$ $x = \sqrt{32}$ $x = \sqrt{16}\sqrt{2}$ $x = 4\sqrt{2}$	(16) $x^{\frac{2}{5}} = 4$
$x^{\frac{2}{3}} = \frac{4}{9}$ $\left(x^{\frac{2}{3}}\right)^{\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}}$ $x^{\frac{1}{3}} = \frac{\sqrt{4}}{\sqrt{9}}$ $\left(x^{\frac{1}{3}}\right)^{3} = \left(\frac{2}{3}\right)^{3}$ $x = \frac{2^3}{3^3}$ $x = \frac{8}{27}$ Note: think about how much harder this would have been if we had started by cubing both sides, rather than square rooting. It would still work,	(17) $x^{\frac{3}{4}} = \frac{1}{27}$ With this question, is it easiest to start by cube rooting each side, or by raising each side to the

but it would have been more difficult.	power 4?

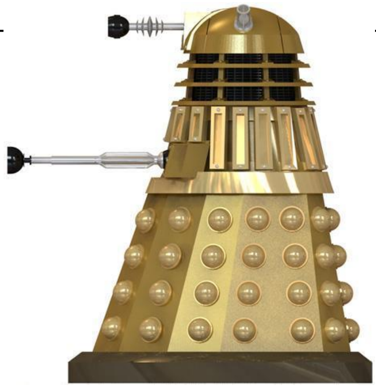
PART A – SECTION 3 : SURDS

A surd is an IRRATIONAL ROOT e.g., $\sqrt{2}$, $\sqrt{3}$, etc., but not $\sqrt[3]{8}$ because $\sqrt[3]{8} = 2$

ANOTHER TOP TIP! When you write a root, make sure that it has a top line which goes over everything in the root, otherwise things can jump out without you noticing!
$\sqrt{}4x$ could mean which is $2x$ or it might mean which is .

NO!!
YES!!

WRONG!!!
Students often make up the rule that a power can be applied to the two terms of a sum separately. Actually, nothing can be done to simplify this expression.



Examples. Tick when you understand.	**Now try exercise 4:** Simplify into the form $a\sqrt{b}$. Tick when correct.
Multiplication and roots: $\sqrt{ab} = \sqrt{a}\sqrt{b}$ $\sqrt{80} = \sqrt{16}\sqrt{5}$ $= 4\sqrt{5}$ ☐	(1) $\sqrt{27} =$ ☐ (2) $\sqrt{45} =$ ☐ (3) $\sqrt{12} =$ ☐ (4) $\sqrt{48} =$ ☐ (5) $\sqrt{75} =$ ☐
Division and roots: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	(6) $\frac{\sqrt{12}}{2} =$ ☐ (7) $\frac{\sqrt{98}}{7} =$ ☐ (8) $\frac{\sqrt{18}}{\sqrt{2}} =$ ☐

$$\sqrt{5\frac{4}{9}} = \sqrt{\frac{49}{9}}$$
$$= \frac{\sqrt{49}}{\sqrt{9}}$$
$$= \frac{7}{3}$$

(9) $\dfrac{\sqrt{27}}{\sqrt{3}} =$

Can you see that top heavy fractions are much nicer than mixed numbers or decimals!

FYING SURDS

ing ... erms. Tick the box when you understand.

$$\sqrt{75} + 2\sqrt{48} - 5\sqrt{12} = \sqrt{(25)(3)} + 2\sqrt{(16)(3)} - 5\sqrt{(4)(3)}$$
$$= \sqrt{25}\sqrt{3} + 2\sqrt{16}\sqrt{3} - 5\sqrt{4}\sqrt{3}$$
$$= 5\sqrt{3} + 2(4)\sqrt{3} - 5(2)\sqrt{3}$$
$$= 5\sqrt{3} + 8\sqrt{3} - 10\sqrt{3}$$
$$= 3\sqrt{3}$$

Exercise 4 continued. Tick when correct.

(10) $\sqrt{12} + 3\sqrt{75} =$

(11) $\sqrt{200} + \sqrt{18} - 2\sqrt{72} =$

(12) $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80} =$

RATIONALISING THE DENOMINATOR

This means write the fraction differently, so there is no surd on the bottom.

TYPE 1 Examples: Multiplying the top and bottom by the surd on the bottom. Tick when understood.	**Exercise 5:** Rationalise the denominators and write in the form $a\sqrt{b}$ (where *a* is usually a fraction). Tick when correct.
$\frac{1}{\sqrt{3}}$ $= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $= \frac{\sqrt{3}}{3}$ $= \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{1}\right)$ $= \frac{1}{3}\sqrt{3}$	(1) $\frac{1}{\sqrt{2}} =$ (2) $\frac{1}{\sqrt{7}} =$
	(3) $\frac{7}{\sqrt{5}} =$

$\frac{1}{4\sqrt{2}}$ $=\frac{\sqrt{2}}{4\sqrt{2}\sqrt{2}}$ $=\frac{\sqrt{2}}{4(2)}$ $=\left(\frac{1}{8}\right)\left(\frac{\sqrt{2}}{1}\right)$ $=\frac{1}{8}\sqrt{2}$ $\boxed{}$	
	(4) $\frac{\sqrt{2}}{3\sqrt{3}} =$ $\boxed{}$

If the denominator is a sum or difference, you can use the clever technique of multiplying top and bottom by the 'opposite' of the denominator to create a difference of two squares on the bottom:

$$(a-b)(a+b)=a^2-b^2$$

TYPE 2 Examples. Multiply top and bottom by the 'opposite' of the bottom. Follow the example carefully then try to do it yourself. Tick when understood.	**Now try Exercise 6:** Rationalise the denominators and write in the form $a+b\sqrt{c}$. Tick when correct.

The bottom is $1+\sqrt{3}$ so we multiply top and bottom by $1-\sqrt{3}$

$$\frac{1}{1+\sqrt{3}} = \frac{(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$
$$= \frac{1-\sqrt{3}}{(1)^2-(\sqrt{3})^2}$$
$$= \frac{1-\sqrt{3}}{1-3}$$
$$= \frac{1-\sqrt{3}}{-2}$$
$$= \frac{1}{-2} - \frac{\sqrt{3}}{-2}$$
$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}$$
$$= -\frac{1}{2} + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{1}\right)$$
$$= -\frac{1}{2} + \frac{1}{2}\sqrt{3}$$

Do you recognise this step from exercise 3?

(1) $\frac{1}{1+\sqrt{2}} =$

The bottom is $4-\sqrt{2}$ so we multiply top and bottom by $4+\sqrt{2}$

$$\frac{3}{4-\sqrt{2}} = \frac{3(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})}$$
$$= \frac{3(4+\sqrt{2})}{(4)^2-(\sqrt{2})^2}$$
$$= \frac{3(4+\sqrt{2})}{16-2}$$
$$= \frac{12+3\sqrt{2}}{14}$$
$$= \frac{12}{14} + \frac{3\sqrt{2}}{14}$$
$$= \frac{6}{7} + \left(\frac{3}{14}\right)\left(\frac{\sqrt{2}}{1}\right)$$
$$= \frac{6}{7} + \frac{3}{14}\sqrt{2}$$

Important step!

(2) $\frac{5}{1-\sqrt{3}} =$

PART A MINI-TEST

So, you've completed all the exercises in part A. Well done!

The important question now is whether you have really learned the techniques in part A. To find out, use this mini-test (in exam conditions); then mark it yourself, using the answers at the back of the booklet, and give yourself a score. You should aim for 25/25 (of course) but anything less than 15/25 should be a worry. Each question number comes from that number exercise. **Go back to the exercises containing the questions you got wrong** then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes. No Calculator allowed.
Good Luck!

1 (a) Write $\frac{3x}{4} \times 5$ as a single fraction

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction

2 (a) Evaluate $32^{\frac{3}{5}}$

(b) Evaluate $9^{-\frac{1}{2}}$

3 (a) Write $\frac{3}{2\sqrt{x}}$ in the form αx^n

(b) Write $\frac{2\sqrt{x}+4}{x^2}$ in the form $\alpha x^n + \beta x^m$

(c) Solve the equation $x^{-\frac{2}{3}} = 9$

4 (a) Simplify $\sqrt{45}$

(b) Simplify $\frac{\sqrt{12}}{2}$

(c) Simplify $\sqrt{200} + \sqrt{18} - 2\sqrt{72}$

5 Rationalise the denominator of $\frac{7}{\sqrt{5}}$ leaving your answer in the form $a\sqrt{5}$

6 Rationalise the denominator of $\frac{1}{1+\sqrt{2}}$

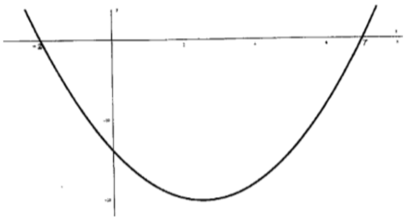
MARK YOUR TEST USING THE SOLUTIONS AT THE BACK OF THE BOOKLET AND PUT YOUR SCORE HERE :   /25

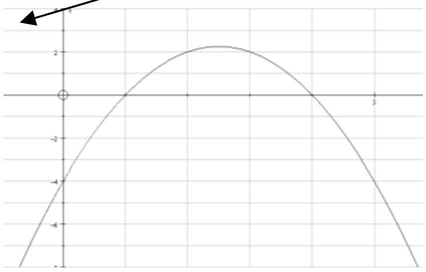
PART B - QUADRATICS

You should know what a 'quadratic' is but in order to start AS you need to REALLY understand and be able to use quadratics. You need to be able to manipulate quadratic expressions by factorising and completing the square and you need to be able to solve quadratic equations using 3 different methods.

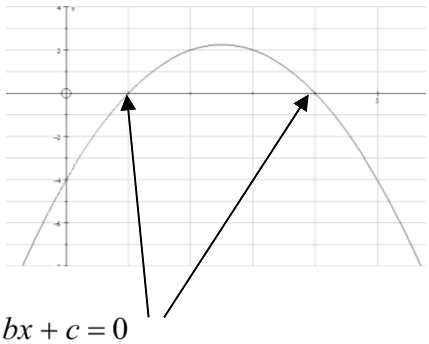
A **QUADRATIC EXPRESSION** is just some algebra written in the form $ax^2 + bx + c$. The numbers a, b and c can be anything you like (b and c could even be zero!).

A **QUADRATIC GRAPH** looks like this depending on whether a is positive or negative:





A **QUADRATIC EQUATION** can always be rearranged to make the right hand side equal to zero, that it is, in the form $ax^2 + bx + c = 0$. The solutions can be seen (where the graph crosses the x-axis). Normally, you would expect there to be two possible answers, as in the graphs above.



Solutions to the equation $ax^2 + bx + c = 0$

Of course, if the quadratic graph is totally above or below the x axis then it will never cross the x axis. In these cases, the quadratic equation has no solutions. Or, possibly, the quadratic graph might just sit on the x axis rather than crossing it, in which case the quadratic equation will only have one solution (called a repeated root).

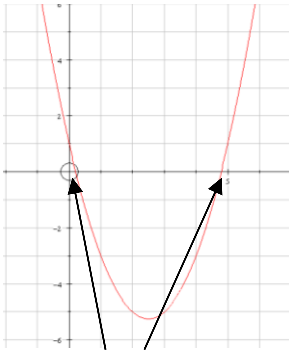
How can we solve the equation $2x^2 + 6x = 8$?

First, get everything on the left hand side so it equals zero…… $2x^2 + 6x - 8 = 0$.

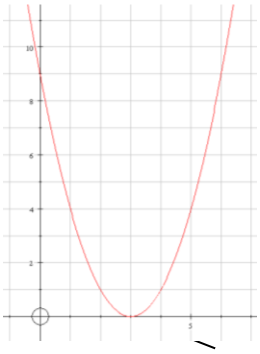
You are now ready to solve the equation – if it can be solved….. This quadratic might have 2 solutions like in the picture above, it might have one solution or it might have no solutions. Over the next few pages, you will first practise working out whether it has none, one or two solutions. Then, you will practise finding the solutions (if they exist!) by three different methods.

PART B – SECTION 1 : THE DISCRIMINANT

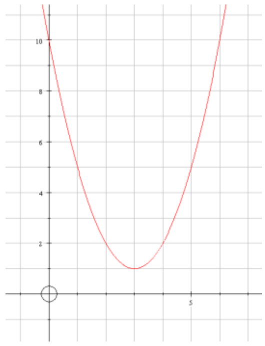
All quadratic graphs cross the y-axis. The y-intercept is the value of the quadratic when $x = 0$. The behaviour on the x-axis is a bit more complicated. Some quadratic graphs cross the x-axis twice, giving two solutions to the equation $ax^2 + bx + c = 0$. Other quadratics simply 'sit' on the x axis, so they only have one solution to the equation $ax^2 + bx + c = 0$. There are also some quadratics which don't cross the x axis at all so these quadratics have no solutions to the equation $ax^2 + bx + c = 0$.



Two distinct roots



One repeated root
(two equal roots)



No real roots

The solutions of an equation, i.e. the places where the graph crosses the x-axis, are called the **roots of the equation**.

We know that the solutions to a quadratic equation are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What could go wrong? Why do we sometimes get two solutions, sometimes one solution and sometimes no solutions?! The answer lies inside the square root sign.

$\underline{b^2 - 4ac > 0}$ (positive)

Everything is fine. We square root $b^2 - 4ac$ and get two solutions using the quadratic formula.

$\underline{b^2 - 4ac = 0}$

If $b^2 - 4ac = 0$ then $\sqrt{b^2 - 4ac} = 0$ so in this case $x = \dfrac{-b \pm 0}{2a} = \dfrac{-b}{2a}$. Just one (repeated) solution.

$\underline{b^2 - 4ac < 0}$ (negative)

If $b^2 - 4ac < 0$ have a problem. We can't square root a negative number so we are stuck. That is why, in this situation, there are no solutions.

$b^2 – 4ac$ is called the DISCRIMINANT of the quadratic because it helps us to discriminate between the quadratics with no roots, quadratics with one repeated root and quadratics with two roots.

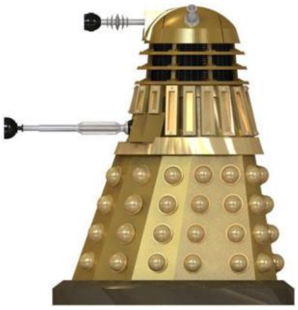
Go back to page 4 of this booklet and look at the quadratics at the bottom of the page. Does it make sense to you that you can see whether the discriminant is positive, negative or zero by looking at the graph of the quadratic? How many times does it cross the x-axis?

It is important to remember that in the discriminant ($b^2 - 4ac$), a represents the amount of x^2 in your quadratic, b represents the amount of x in your quadratic and c represents everything else in your quadratic (ie the numbers). **Don't let yourself get muddled if the quadratic is written in a different order!**

Exercise 7

Write down the discriminant of each of these quadratics and hence state whether each one has two roots, one repeated root or no roots. Tick when co

> Note: make sure that you square all of b!
>
> If b is – 6 then b^2 is $(-6)^2 = 36$ (**NOT** –36)
>
> If b is $2k$ then $b^2 = (2k)^2 = 4k^2$ (**NOT** $2k^2$)



Quadratic	Value of Discriminant	Circle the number of roots
EXAMPLE (1) $x^2 + 8x + 7$	$(\ 8\ )^2 - 4(\ 1\ )(\ 7\ ) = 36$ $36 > 0$	None One Repeated Two ☐
(2) $3x + x^2 - 2$	$(\quad)^2 - 4(\quad)(\quad) =$	None One Repeated Two ☐
(3) $x^2 + 3$	$(\quad)^2 - 4(\quad)(\quad) =$	None One Repeated Two ☐
(4) $2x^2 + 3 - 6x$	$(\quad)^2 - 4(\quad)(\quad) =$	None One Repeated Two ☐
(5) $x - x^2$	$(\quad)^2 - 4(\quad)(\quad) =$	None One Repeated Two ☐
(6) $x^2 - 6x + 9$	$(\quad)^2 - 4(\quad)(\quad) =$	None One Repeated Two ☐

Part B - Section 2 : Factorising Quadratics

… Using the difference of two squares: $(a)^2 - (b)^2 = (a - b)(a + b)$

	Exercise 8 Factorise the following	Tick when correct
<u>Example 1</u> $x^2 - 9 = (x - 3)(x + 3)$	(1) $x^2 - 1$	
	(2) $4x^2 - 9$	
<u>Example 2</u> $9x^2 - 16 = (3x)^2 - (4)^2$ $= (3x - 4)(3x + 4)$	(3) $49 - x^2$	
	(4) $2x^2 - 8$	
	(5) $x^2 - 16$	
<u>Example 3</u> $8x^2 - 2 = 2(4x^2 - 1)$ $= 2((2x)^2 - 1^2)$ $= 2(2x - 1)(2x + 1)$	(6) $9x^2 - 1$	
	(7) $36 - 25x^2$	
	(8) $9x^2 - 36$	

Exercise 9

Factorise the following quadratics. Remember to expand out to check your answers. The first one has been completed for you. Tick when correct!

(1) $x^2 - 2x - 15$	(2) $6x^2 - 3x$	(3) $x^2 - 5x - 6$
$= (x - 3)(x + 5)$ Check: $(x - 3)(x + 5)$ $= x^2 - 3x + 5x - 15$ $= x^2 - 2x - 15$		
(4) $x^2 + x - 6$	(5) $2x^2 + 6x$	(6) $x^2 - 6x - 16$

Exercise 10 Factorise the following. Don't forget to expand out to check your answers. Tick when correct.

(1) $2x^2 + 5x + 2$	(2) $3x^2 - 8x + 4$
(3) $2x^2 + 7x + 6$	(4) $3x^2 - 13x - 10$
(5) $2x^2 + 9x - 5$	(6) $2x^2 - 11x + 12$

PART B – SECTION 3 : COMPLETING THE SQUARE

Completing the square is a bit like factorising. It doesn't change the quadratic but it changes the way the quadratic expression is written. When we factorise, we change $x^2 + bx + c$ into $(x - p)(x - q)$ by finding p and q When we complete the square, we change $x^2 + bx + c$ into $(x + p)^2 + q$ by finding p and q

$$\boldsymbol{x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c}$$

	Exercise 11 Complete the square of the following quadratics.	
Example Express $x^2 + 6x + 11$ in the completed square form $(x+p)^2 + q$. $x^2 + 6x + 11 = (x+3)^2 - (3)^2 + 11$ $= (x+3)^2 - 9 + 11$ $= (x+3)^2 + 2$ Tick when understood.	**(1)** $\boldsymbol{x^2 + 8x + 7}$	(2) $x^2 - 2x - 15$
Example 2 Express $x^2 - 10x + 13$ in the completed square form $(x+p)^2 + q$. $x^2 - 10x + 13 = (x + -5)^2 - (-5)^2 + 13$ $= (x-5)^2 - 25 + 13$ $= (x-5)^2 -\text{`}12$	(3) $x^2 + 6x + 10$	(4) $x^2 - 10x + 9$
(5) $x^2 + 12x + 100$	(6) $x^2 + 2x - 6$	(7) $x^2 + 6x - 5$

PART B – SECTION 4 - SOLVING QUADRATICS

There are 3 ways to solve a quadratic equation: by factorising, by using the quadratic formula or by completing the square.

- Factorising uses the fact that if 2 things multiply together to make zero then one of them MUST be zero. You can't always factorise a quadratic, even if it has solutions.
- The quadratic formula will always give you the solutions, so long as there are some!
- Completing the square allows you to simply rearrange the quadratic to find x. If there are solutions to the quadratic equation then completing the square will always work.

Example - Factorising

$2x^2 - 5x + 3 = 0$

Factorising gives: $(2x - 3)(x - 1) = 0$

so either $2x - 3 = 0$ or $x - 1 = 0$

$2x = 3$

$\therefore x = {}^3/_2$ or $x = 1$

This means that the graph of the quadratic function $f(x) = 2x^2 - 5x + 3$ crosses the x axis at ${}^3/_2$ and 1.

Tick when understood

FACTORISING

Exercise 12 Solve the following quadratic equations by factorising. Tick when correct.

(1) $x^2 + 11x + 28 = 0$	(2) $x^2 + 3x = 0$	(3) $2x^2 + 3x - 14 = 0$

THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Now try Exercise 13: Solve the following quadratic equations using the quadratic formula, leaving your answers in the form $x = A \pm B\sqrt{C}$ as in the example on the left.

To solve the quadratic equation $ax^2 + bx + c = 0$ you can use the quadratic formula above.

Then you will need to rearrange these answers into the form

$$x = A \pm B\sqrt{C}$$

<u>Example – Using the formula</u>

Solve $x^2 + 3x + 1 = 0$

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2 \times 1} \\ &= \frac{-3 \pm \sqrt{9-4}}{2} \\ &= \frac{-3 \pm \sqrt{5}}{2} \\ &= -\frac{3}{2} \pm \frac{\sqrt{5}}{2} \\ &= -\frac{3}{2} \pm \left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right) \\ &= -\frac{3}{2} \pm \frac{1}{2}\sqrt{5} \end{aligned}$$

Important step!

Tick when understood ☐

(1) $2x^2 + 4x + 1 = 0$

(2) $x^2 - 7x + 9 = 0$

To solve the quadratic equation $ax^2 + bx + c = 0$, you can complete the square and then rearrange the equation. The answers will come out nicely in the form you want: $x = A \pm B\sqrt{C}$. In the examples below, we have shown every step to help you follow what is happening.

<table>
<tr><td>Example.</td><td>Now try Exercise 14 Solve this quadratic by completing the square. Tick when correct.</td></tr>
<tr><td>

Solve $x^2 + 3x + 1 = 0$ by completing the square

$$\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+1=0$$

First complete the square, then expand out the (half *b*)^2 bit (remember to square the top AND bottom of the fraction) then add it to *c*

$$\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+\frac{4}{4}=0$$

$$\left(x+\frac{3}{2}\right)^2-\frac{5}{4}=0$$

$$\left(x+\frac{3}{2}\right)^2=\frac{5}{4}$$

Put the number on the right hand side then square root both sides, remembering to add the ± sign!

$$x+\frac{3}{2}=\pm\sqrt{\frac{5}{4}}$$

$$x+\frac{3}{2}=\pm\frac{\sqrt{5}}{\sqrt{4}}$$

$$x+\frac{3}{2}=\pm\left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right)$$

Finally move the 'half of *b*' to the other side so it says *x* = ,

$$x=-\frac{3}{2}\pm\frac{1}{2}\sqrt{5}$$

$$x=-\frac{3}{2}+\frac{1}{2}\sqrt{5}\ \ or\ \ x=-\frac{3}{2}-\frac{1}{2}\sqrt{5}$$

Tick when understood ☐

</td><td>

(1) $x^2 + 2x - 6 = 0$

☐

</td></tr>
<tr><td>Example – Using the 'Completed Square' to solve a quadratic.</td><td>Exercise 14 continued Solve this quadratic by completing the square. Tick when correct.</td></tr>
</table>

Solve $x^2 - x = 0$ by completing the square

$$\left(x - \frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 = 0$$

First complete the square, then expand out the (half b)^2 bit. In this question, c = 0

$$\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = 0$$

Put the number on the right hand side then square root both sides, remembering to add the ± sign!

$$\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{1}{4}}$$

Remember: the square root of a fraction is the square root of the top, over the square root of the bottom.

$$x - \frac{1}{2} = \pm\frac{\sqrt{1}}{\sqrt{4}}$$

$$x - \frac{1}{2} = \pm\frac{1}{2}$$

Finally move the 'half of b' to the other side so it says 'x = '.

$$x = \frac{1}{2} - \frac{1}{2} \quad or \quad x = \frac{1}{2} + \frac{1}{2}$$

$$x = 0 \qquad or \quad x = 1$$

Tick when understood

(2) $x^2 + 6x - 5 = 0$

PART B MINI - TEST

So, you've completed all the exercise in part B. Well done! The important question is whether you have really learned these techniques. To find out, use this mini test (in exam conditions) then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong and then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes. No Calculator allowed.
Good Luck!

7 Evaluate the discriminant of the quadratic $y = 2x^2 + 3 - 6x$ and hence state the number of roots of the equation $2x^2 + 3 - 6x = 0$

8 Factorise the quadratic $y = 4x^2 - 9$ using the difference of two squares.

9 Factorise the quadratic $y = 2x^2 + 6x$

10 Factorise the quadratic $y = 3x^2 - 13x - 10$

11 Write the quadratic $y = x^2 + 8x + 7$ in completed square form.

12 Solve the equation $x^2 + 3x = 0$ by factorising.

13 Solve the equation $2x^2 + 4x + 1 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.

14 Solve the equation $x^2 + 6x - 5 = 0$ by rearranging the completed square, leaving the answer(s) in surd form.

Quadratic formula: $$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Completed square: $$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

MARK YOUR TEST USING THE SOLUTIONS AT THE BACK OF THE BOOKLET AND PUT YOUR SCORE HERE /20

ARE YOU READY FOR AS? MOCK TEST

In order to be ready to start AS Maths, you need to be confident with the techniques in this booklet. **In the second week** of the course, we will give you a test like this one to check that you are well prepared and ready to start AS Maths. Try the following mock test in exam conditions; USE LINED PAPER ONLY. Your teacher will mark this work and give you feedback on the following:

Understanding of the questions.
Ability to show clear workings.
Accuracy of your answers.

Time: 1 hour. No Calculator allowed.
Good Luck!

1(a) Write $\frac{3x}{2} \div 5$ as a single fraction.

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction.

2(a) Evaluate $16^{-\frac{7}{4}}$

(b) Evaluate $4^{\frac{5}{2}}$

3(a) Write $\frac{2+\sqrt{x}}{\sqrt{x}}$ in the form $\alpha x^n + \beta x^m$

(b) Solve the equation $x^{\frac{3}{4}} = \frac{1}{27}$

4(a) Simplify $\sqrt{48}$

(b) Simplify $\frac{\sqrt{18}}{\sqrt{2}}$

(c) Simplify $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}$

5(a) Rationalise the denominator of $\frac{\sqrt{2}}{3\sqrt{3}}$
leaving your answer in the form $a\sqrt{6}$

(b) Rationalise the denominator of $\frac{1}{\sqrt{2}}$

6 Rationalise the denominator of $\overline{1-\sqrt{3}}$

Don't forget to bring your completed test with clea
Maths lesson.

7 Evaluate the discriminant of the quadratic $y = x - x^2$ and hence state the number of roots of the equation $x - x^2 = 0$.

8 Factorise the quadratic $y = 2x^2 - 8$ using the difference of two squares.

9 Factorise the quadratic $y = 6x^2 - 3x$.

10 Factorise the quadratic $y = 2x^2 - 11x + 12$.

11 Write the quadratic $y = x^2 - 6x - 16$ in completed square form.

12 Solve the equation $2x^2 + 3x - 14 = 0$ by factorising.

13 Solve the equation $x^2 - 7x + 9 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.

14 Solve the equation $x^2 + 2x - 6 = 0$ by rearranging the completed square, leaving the answer(s) in surd form.

15 Solve the inequality $x^2 - 8x + 15 < 0$.

ANSWERS

Exercise 1

(1) $\frac{15x}{4}$	(2) $\frac{2x+3}{x^2}$	(3) $\frac{3x}{10}$

Exercise 2

(1) $\frac{1}{64}$	(2) $\frac{1}{3}$	(3) $\frac{1}{3}$	(4) 32	(5) 8	(6) $\frac{1}{128}$

Exercise 3

(1) $\frac{1}{5}x$	(2) $\frac{3}{2}x^{-\frac{1}{2}}$	(3) $\frac{1}{3}x^{-\frac{3}{2}}$	(4) $2x^{\frac{2}{3}}$
(5) $2x^{-\frac{3}{2}}+4x^{-2}$	(6) $\frac{2}{3}x^{-1}+\frac{4}{3}x^{-2}$	(7) $\frac{1}{4}x^{-3}-x^{-2}$	(8) $x^{-1}-4x^{-\frac{1}{2}}$
(9) $x^{\frac{3}{2}}-3x^{-\frac{1}{2}}$	(10) $x^{-1}-2x^{-2}$	(11) $2x^{-\frac{1}{2}}+1$	(12) $\frac{1}{2}+x^{-1}$
(13) $\frac{1}{3}x^{-\frac{3}{2}}+2x^{-2}$	(14) $2x^{-1}-x^{-2}$		
(15) $x=\pm\frac{1}{27}$	(16) $x=\pm 32$	(17) $x=\frac{1}{81}$	

Exercise 4

(1) $3\sqrt{3}$	(2) $3\sqrt{5}$	(3) $2\sqrt{3}$	(4) $4\sqrt{3}$	(5) $5\sqrt{3}$	(6) $\sqrt{3}$
(7) $\sqrt{2}$	(8) 3	(9) 3	(10) $17\sqrt{3}$	(11) $\sqrt{2}$	(12) $-4\sqrt{5}$

Exercise 5

(1) $\frac{1}{2}\sqrt{2}$	(2) $\frac{1}{7}\sqrt{7}$	(3) $\frac{7}{5}\sqrt{5}$	(4) $\frac{1}{9}\sqrt{6}$

Exercise 6

(1) $-1+\sqrt{2}$	(2) $-\frac{5}{2}-\frac{5}{2}\sqrt{3}$

Exercise 7

(1) 36, two	(2) 17, two	(3) – 12, none
(4) 12, two	(5) 1, two	(6) 0, one repeated

Exercise 8

(1) $(x-1)(x+1)$	(2) $(2x-3)(2x+3)$	(3) $(7-x)(7+x)$
(4) $2(x-2)(x+2)$	(5) $(x-4)(x+4)$	(6) $(3x-1)(3x+1)$
(7) $(6-5x)(6+5x)$	(8) $9(x-2)(x+2)$	

Exercise 9

(1) $(x+3)(x-5)$	(2) $3x(2x-1)$	(3) $(x-6)(x+1)$
(4) $(x-2)(x+3)$	(5) $2x(x+3)$	(6) $(x-8)(x+2)$

Exercise 10

(1) $(2x+1)(x+2)$	(2) $(3x-2)(x-2)$	(3) $(2x+3)(x+2)$
(4) $(3x+2)(x-5)$	(5) $(2x-1)(x+5)$	(6) $(2x-3)(x-4)$

Exercise 11

(1) $(x+4)^2 - 9$	(2) $(x-1)^2 - 16$	(3) $(x+3)^2 + 1$
(4) $(x-5)^2 - 16$	(5) $(x+6)^2 + 64$	(6) $(x+1)^2 - 7$
(7) $(x+3)^2 - 14$		

Exercise 12

(1) $x = -7$ or -4	(2) $x = 0$ or -3	(3) $x = -{}^{7}/_{2}$ or 2

Exercise 13

(1) $x = -1 \pm \frac{1}{2}\sqrt{2}$

(2) $x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$

Exercise 14

(1) $x = -1+\sqrt{7}$ or $x = -1-\sqrt{7}$	(2) $x = -3+\sqrt{14}$ or $x = -3-\sqrt{14}$

Part A Mini Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!) , 1 mark for the correct method (but made a mistake) and 0 marks for doing it totally wrong! Give yourself a bonus mark if you got (6b) correct ☺. The test is out of 25 and **anything below 15/25 is worrying; you must go back to the exercises and try to master the techniques, using the tips on page 2 of the booklet for help.**

1 (a) $\frac{3x}{4}\times 5 = \left(\frac{3x}{4}\right)\left(\frac{5}{1}\right) = \frac{15x}{4}$

1 (b) $\frac{2}{x}+\frac{3}{x^2} = \frac{2x}{x^2}+\frac{3}{x^2} = \frac{2x+3}{x^2}$

2 (a) $32^{\frac{3}{5}} = \left(\sqrt[\frac{1}{5}]{32}\right)^3 = 2^3 = 8$

2 (b) $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

3 (a) $\frac{3}{2\sqrt{x}} = \left(\frac{3}{2}\right)\left(\frac{1}{\sqrt{x}}\right) = \frac{3}{2}x^{-\frac{1}{2}}$

(b) $\frac{2\sqrt{x}+4}{x^2} = \frac{2\sqrt{x}}{x^2}+\frac{4}{x^2} = \left(\frac{2}{1}\right)\left(\frac{\sqrt{x}}{x^2}\right)+\left(\frac{4}{1}\right)\left(\frac{1}{x^2}\right) = 2x^{\frac{1}{2}-2}+4x^{-2} = 2x^{-\frac{3}{2}}+4x^{-2}$

(c)

$$x^{-\frac{2}{3}} = 9$$

$$x^{\frac{2}{3}} = \frac{1}{9}$$

$$x^{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{9}}$$

$$x^{\frac{1}{3}} = \pm\frac{1}{3}$$

$$x = \left(\pm\frac{1}{3}\right)^3$$

$$x = \pm\frac{1}{27}$$

4 (a) $\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

(b) $\frac{\sqrt{12}}{2} = \frac{\sqrt{4}\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

(c) $\sqrt{200}+\sqrt{18}-2\sqrt{72} = \sqrt{100}\sqrt{2}+\sqrt{9}\sqrt{2}-2\sqrt{36}\sqrt{2} = 10\sqrt{2}+3\sqrt{2}-12\sqrt{2} = \sqrt{2}$

5 $\frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}}{5} = \left(\frac{7}{5}\right)\left(\frac{\sqrt{5}}{1}\right) = \frac{7}{5}\sqrt{5}$

6 $\frac{1}{1+\sqrt{2}} = \frac{1\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} = \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = \frac{1}{-1} - \frac{\sqrt{2}}{-1} = -1+\sqrt{2}$

Part B Mini Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!), 1 mark for the correct method (but made a mistake) and 0 marks for doing it totally wrong! Give yourself 2 bonus marks if you got (7) correct and 2 bonus marks if you got (8) correct ☺.
The test is out of 20 and **anything below 12/20 is worrying; you must go back to the exercises and try to master the techniques, using the tips on page 2 of the booklet for help.**

7 $b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$. 12 > 0 (hence the equation has 2 distinct roots).

8 $4x^2 - 9 = (2x-3)(2x+3)$

9 $2x^2 + 6x = 2x(x+3)$

10 $3x^2 - 13x - 10 = (3x+2)(x-5)$

11 $x^2 + 8x + 7 = (x+4)^2 - (4)^2 + 7 = (x+4)^2 - 9$

12

$$\begin{aligned}
&x^2 + 3x = 0 \\
&x(x+3) = 0 \\
&x = 0,\ \ x+3 = 0 \\
&x = 0,\ \ x = -3
\end{aligned}$$

13

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{16-8}}{4} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4}{4} \pm \frac{\sqrt{8}}{4}$$

$$= -1 \pm \frac{\sqrt{4}\sqrt{2}}{4} = -1 \pm \left(\frac{2}{4}\right)\left(\frac{\sqrt{2}}{1}\right) = -1 \pm \frac{1}{2}\sqrt{2}$$

14

$$\begin{aligned}
&x^2 + 6x - 5 = 0 \\
&(x+3)^2 - (3)^2 - 5 = 0 \\
&(x+3)^2 - 14 = 0 \\
&(x+3)^2 = 14 \\
&x + 3 = \pm\sqrt{14} \\
&x = -3 \pm \sqrt{14}
\end{aligned}$$