



Department
for Education

The national curriculum in England

Maths

Mathematics

Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Information and communication technology (ICT)

Calculators should not be used as a substitute for good written and mental arithmetic. In secondary schools, teachers should use their judgement about when ICT tools should be used.

Spoken language

The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.

Schools are not required by law to teach the example content in [square brackets] or the content indicated as being ‘non-statutory’.

Introduction

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programme of study for key stage 3 is organised into apparently distinct domains, but pupils should build on key stage 2 and connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge in science, geography, computing and other subjects.

The expectation is that the majority of pupils will move through the programme of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils’ understanding and their readiness to progress. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on.

Attainment targets

By the end of key stage 3, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.

Working mathematically

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and

equations and graphs]

- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non routine problems.

Subject content

Number

Pupils should be taught to:

- understand and use place value for decimals, measures and integers of any size
- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- recognise and use relationships between operations including inverse operations
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or zero
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
- define percentage as ‘number of parts per hundred’, interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators
- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a

number of decimal places or significant figures]

- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- appreciate the infinite nature of the sets of integers, real and rational numbers.

Algebra

Pupils should be taught to:

- use and interpret algebraic notation, including:
 - ab in place of $a \times b$
 - $3y$ in place of $y + y + y$ and $3 \times y$
 - a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$
 - b in place of $a \div b$
 - coefficients written as fractions rather than as decimals
 - brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
 - collecting like terms
 - multiplying a single term over a bracket
 - taking out common factors
 - expanding products of two or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)

- work with coordinates in all four quadrants
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the n th term
- recognise geometric sequences and appreciate other sequences that arise.

Ratio, proportion and rates of change

Pupils should be taught to:

- change freely between related standard units [for example time, length, area, volume/capacity, mass]
- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics

- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, unit pricing and density to solve problems.

Geometry and measures

Pupils should be taught to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- identify properties of, and describe the results of, translations, rotations and reflections applied to given figures
- identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles

- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs
- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D
- interpret mathematical relationships both algebraically and geometrically.

Probability

Pupils should be taught to:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.

Statistics

Pupils should be taught to:

- describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs.

Introduction

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programme of study for key stage 4 is organised into apparently distinct domains, but pupils should develop and consolidate connections across mathematical ideas. They should build on learning from key stage 3 to further develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge wherever relevant in other subjects and in financial contexts.

The expectation is that the majority of pupils will move through the programme of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

This programme of study specifies:

- the mathematical content that should be taught to all pupils, in standard type; and
- additional mathematical content to be taught to more highly attaining pupils, in **bold** type and braces { }.

Together, the mathematical content set out in the key stage 3 and key stage 4 programmes of study covers the full range of material contained in the GCSE Mathematics qualification. Wherever it is appropriate, given pupils' security of understanding and readiness to progress, pupils should be taught the full content set out in this programme of study.

Working mathematically

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots {**and fractional indices**}
- select and use appropriate calculation strategies to solve increasingly complex problems, including exact calculations involving multiples of π {**and surds**}, use of standard form and application and interpretation of limits of accuracy

- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions, **{and expressions involving surds and algebraic fractions}**
- extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations and inequalities
- move freely between different numerical, algebraic, graphical and diagrammatic representations, including of linear, quadratic, reciprocal, **{exponential and trigonometric}** functions
- use mathematical language and properties precisely.

Reason mathematically

- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically
- extend their ability to identify variables and express relations between variables algebraically and graphically
- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **{and proofs}**
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- assess the validity of an argument and the accuracy of a given way of presenting information.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts

- make and use connections between different parts of mathematics to solve problems
- model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions
- select appropriate concepts, methods and techniques to apply to unfamiliar and non routine problems; interpret their solution in the context of the given problem.

Subject content

Number

In addition to consolidating subject content from key stage 3, pupils should be taught

to: ▪ apply systematic listing strategies, **{including use of the product rule for counting}** ▪ **{estimate powers and roots of any given positive number}**

- calculate with roots, and with integer **{and fractional}** indices
- calculate exactly with fractions, **{surds}** and multiples of π ; **{simplify surd**

expressions involving squares [for example

denominators}

= rationalise = × = 2 and

- calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer ▪ **{change recurring decimals into their corresponding fractions and vice versa}** ▪ identify and work with fractions in ratio problems

- apply and interpret limits of accuracy when rounding or truncating, **{including upper and lower bounds}**.

Algebra

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- simplify and manipulate algebraic expressions (including those involving surds **{and algebraic fractions}**) by:
 - factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **{factorising quadratic expressions of the form**

$$ax^2 + bx + c$$

- simplifying expressions involving sums, products and powers, including the laws of indices
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the ‘inverse function’; interpret the succession of two functions as a ‘composite function’}**
 - use the form $y = mx + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically **{and turning points by completing the square}**
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **{the exponential function $y = k^x$ for positive values of k , and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size}**
- **{sketch translations and reflections of the graph of a given function}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
 - **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**
- **{recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point}**

solve quadratic equations **{including those that require rearrangement}** algebraically by factorising, **{by completing the square and by using the quadratic formula}**; find approximate solutions using a graph

solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph

- **{find approximate solutions to equations numerically using iteration}**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- solve linear inequalities in one **{or two}** variable{s}, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a positive rational number **{or a surd}**) **{and other sequences}**
- deduce expressions to calculate the n^{th} term of linear **{and quadratic}** sequences.

Ratio, proportion and rates of change

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)
- convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- understand that X is inversely proportional to Y is equivalent to X is proportional

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:
Y

{construct and} interpret equations that describe direct and inverse proportion

- interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- set up, solve and interpret the answers in growth and decay problems, including compound interest **{and work with general iterative processes}**.

Geometry and measures

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- interpret and use fractional **{and negative}** scale factors for enlargements
- **{describe the changes and invariance achieved by combinations of rotations, reflections and translations}**
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- **{apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}**
- construct and interpret plans and elevations of 3D shapes
- interpret and use bearings
- calculate arc lengths, angles and areas of sectors of circles
 - calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right angled triangles **{and, where possible, general triangles}** in two **{and three}** dimensional figures
- know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° ; know the exact value of $\tan\theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ$ and 60°

- **{know and apply the sine rule,**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
and cosine rule,

$$a^2 = b^2 + c^2 - 2bc \cos A$$
, to find unknown lengths and angles}

- **{know and apply triangle**

$$\text{Area} = \frac{1}{2} ab \sin C$$
to calculate the area, sides or angles of any

$$\text{Area} = \frac{1}{2} ab \sin C$$

- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**.

Probability

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**.

Statistics

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- **{construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}**
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
 - appropriate graphical representation involving discrete, continuous and grouped data, **{including box plots}**
 - appropriate measures of central tendency (including modal class) and spread **{including quartiles and inter-quartile range}**
- apply statistics to describe a population

- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.