

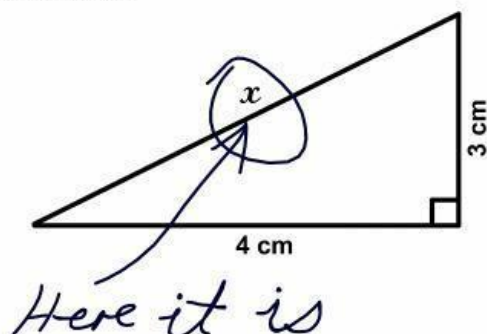
STARTING WITH CONFIDENCE

The last 3 pages of this booklet are very important! Don't forget to complete them!

Name:.....

This booklet has been designed to help you to bridge the gap between GCSE Maths and AS Maths.

3. Find x .



Ocular Trauma - by Wade Clarke ©2005

You need to complete this booklet and bring it with you to your first maths lesson in September. You will be tested on these topics at the start of Year 12 to ensure you have the skills needed to be successful.

The test questions are in the exercises in this booklet and there is an example test for you to use to make sure you are ready!

SIMILARITIES AND DIFFERENCES BETWEEN GCSE AND A LEVEL MATHS

GCSE

Fractions and Decimals are
equally nice and mixed numbers
(like $1\frac{1}{2}$) are ok too

If you're good at maths you can do well without trying

It's the answer that matter most, but you should show working

*A successful student is not one who does
not encounter problems, but one who seeks the
help they need to overcome the problem.*

It's the method that matters, not the answer. Usually you
are given the answer and need to explain the method

A LEVEL

Fractions are MUCH better than decimals
and mixed numbers are *not* nice!

You will do a lot of study
outside of class

When (not if) you get stuck.....

Studying Maths at advanced level is about learning how to solve problems. The first stage of solving a problem is being stuck so you should expect to get stuck while working through this booklet. Some of these topics may seem unfamiliar to you but they are all GCSE level topics and you need to be able to do all these techniques before you start AS Maths.

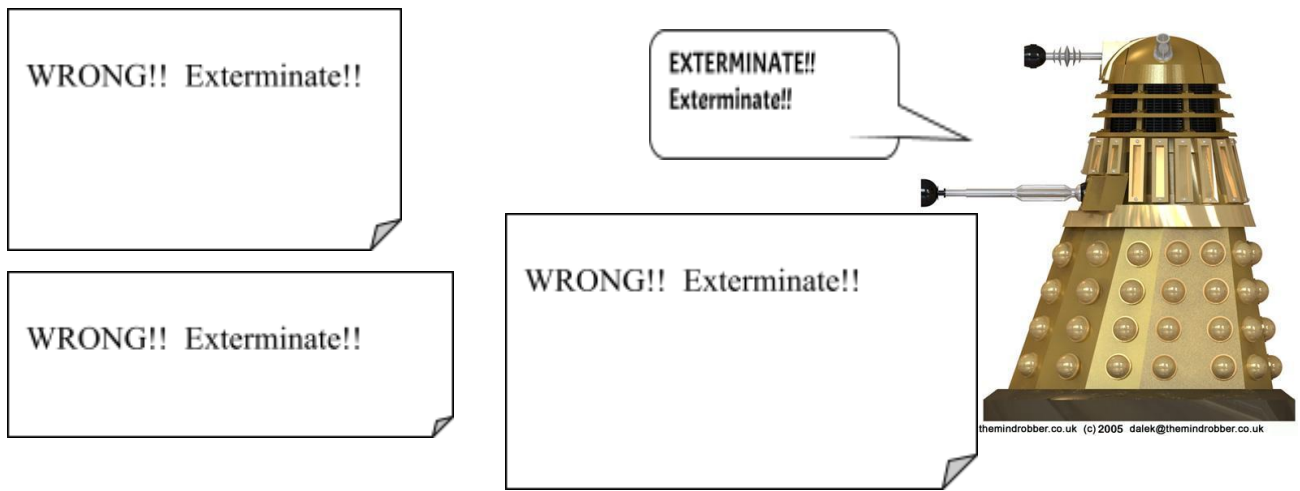
So, when you get stuck...

- **Look again at the examples. Maybe there is one which shows you how to solve your problem?**
- **Have you made a mistake? It might be that your method is correct but you've made an error in your working somewhere.**
- **Try looking up the topic in a GCSE higher tier textbook or revision guide (you can get these from your local library) or look online**

CONTENTS

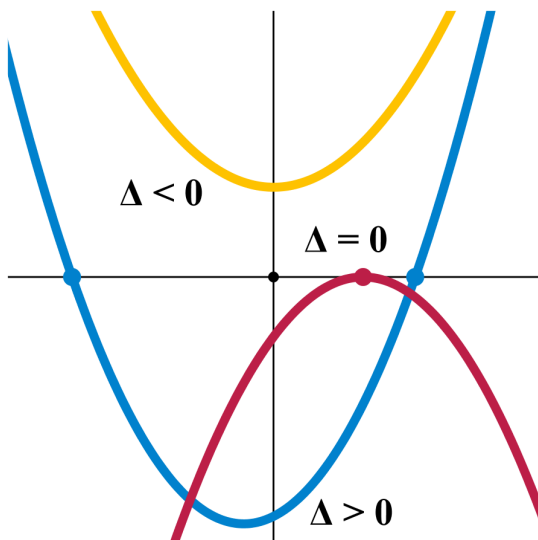
Part A – Learning to Avoid common algebraic ‘Mistakes’.

We all make occasional mistakes when manipulating algebra and learning to make fewer mistakes (and finding the ones you have made!) is an important part of the study of maths at advanced level. However, there are also mistakes that aren't mistakes at all but are actually the result of a deeply held misunderstanding about the laws of algebra. These misunderstandings need to be exterminated as soon as possible. Do you understand why these examples are wrong? **Add them to your table of common mistakes on the back pages and think about how you know they are wrong.**



Part B – Developing Confidence with Quadratics

A quadratic is any algebraic expression with some x^2 bits and some x bits and a number i.e. $ax^2 + bx + c$. In your study of GCSE maths you will have met and learned to solve quadratic equations. In order to cope with the demands of AS Maths you need to be confident working with quadratics and this is something we have found to cause a lot of problems in the transition from GCSE to AS maths. This part of the booklet will outline everything you need to remember about quadratics and give you a chance to practise building your confidence with these important equations.



You should recognise these curves as quadratic curves.

On page 17 of this booklet you will learn what Δ is and how to measure and interpret it for any quadratic ☺

PS. You should be able to complete this entire booklet WITHOUT using a calculator

SUGGESTED STUDY PLAN

Do you feel really confident with all of the A and A* techniques that you learnt at GCSE?

No

We will not have time to cover these techniques in class next year, but you **ARE** required to know them when you start AS. Therefore you need to practise over the Summer. The exercises in this booklet are designed to help you do that. It would be better if you practise little and often, rather than a lot all at once.

Look at the breakdown of topics and approximate timings on the back page and plan your time appropriately.

Work through the exercises in Part A until you are confident with all of the techniques.

On a different day, do the section A mini-test. Did you pass (and do better than last time)?

Yes

Work through the exercises in Part B until you are confident with all of the techniques.

On a different day, do the section B mini-test. Did you pass (and do better than last time)?

Yes

In the last week of the holidays, do the "Are you Ready for AS Test?" (pg 27). Did you score...?

Less than 60%

Go through the exercises again where you are having problems. Attend the support sessions offered between 5th-7th September.

60-80%

Identify the areas where you are making mistakes. Go through the relevant exercises again. Consider attending the support sessions offered between 5th-7th September.

More than 80%

Well done – you have the necessary building blocks in place in order to start AS Maths with confidence.

Yes

Do the mini-tests at the end of Part A (pg 15) and Part B (pg 26). Did you score ...?

Less than 60%

You need to be much more confident with these techniques before September. Work through the whole booklet carefully (again) and use the "When You Get Stuck" tips on page 2 to help you make progress.

60-90%

Pretty good but there are obviously some areas you still need to work on. Identify these sections in the booklet. Go through the examples carefully and do the exercises (again!)

More than 90%

This is a really good score – **well done!** Go over your mistakes. What mistakes did you make? How could you avoid making them in the future? Use the examples and exercises in the booklet to help you.

Finally, make sure you have gone through the booklet and collected together the common mistakes (indicated by daleks). Add them to your table at the back of the booklet together with a correction / explanation.

Part A – Section 1 - FRACTIONS

TOP TIP! Never use a slanted line like this $\frac{3x}{4}$ because the x will try to escape by moving right a bit and growing . It is much harder for the x to escape if you use a horizontal line.

TOP TIP! You will make fewer mistakes if you write things next to each other like $3x$ rather than 3 and x rather than $3x$.

TOP TIP! If you want to multiply a fraction by a number, you can write the number as a fraction by putting it over 1: $\frac{5}{1}$. This avoids the possibility of making the **common mistake** that

Exercise 1 In the spaces available, carry out the following, leaving your answer as a single fraction.

<p>(1) $\frac{3x}{4} \times 5$ (hint: look at top tip 3 if you are not sure about this)</p>	<p>(2) $\frac{2}{x} + \frac{3}{x^2}$ (hint: make the denominators the same by multiplying top and bottom of $\frac{2}{x}$ by x, then add the numerators)</p>	<p>(3) $\frac{3x}{2} \div 5$ (hint: use top tip 3 then remember that dividing by a fraction is the same as multiplying by its reciprocal)</p>
<p>Answers at the back</p>		

Part A – Section 2 - INDICES

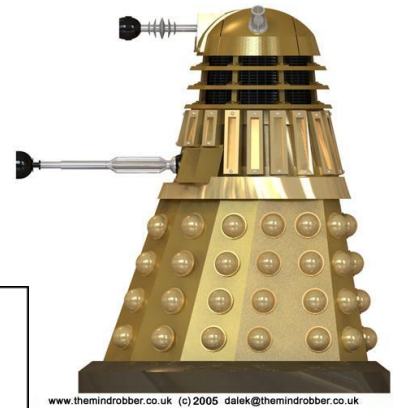
WRONG!!!

Students often think that if there is multiplication in the powers it must correspond to multiplication.

In fact, or .

WRONG!!!

Students often think that if there is addition in the power it must correspond to addition. In fact, .



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Exercise 2

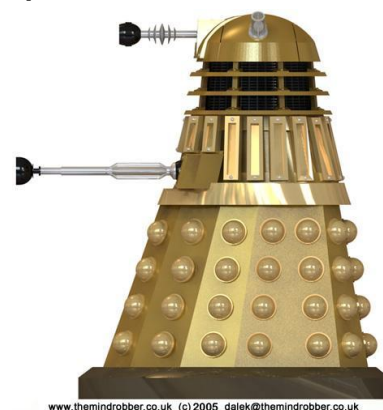
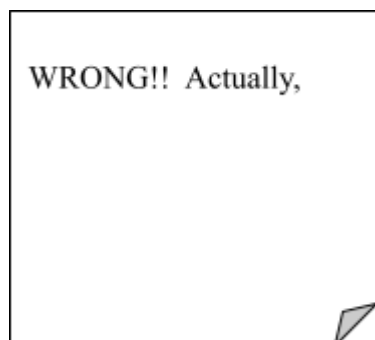
Evaluate the following, tick the boxes when they are correct:

<u>THE RULES OF INDICES</u>		
Rules: $a^m a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^m)^n = a^{mn}$
Also: $(ab)^n = a^n b^n$	$a^0 = 1$	$a^1 = a$
A <u>negative</u> power indicates a <u>reciprocal</u> e.g. 6^{-2} means $\frac{1}{6^2} = \frac{1}{36}$ and 5^{-3} means $\frac{1}{5^3} = \frac{1}{125}$	(1) 2^{-6}	<input type="checkbox"/>
	(2) $9^{-\frac{1}{2}}$	<input type="checkbox"/>
Example: $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = (\sqrt{4})^3 = \boxed{} 8$ Tick the box when you understand!	(3) $81^{\frac{1}{4}}$	<input type="checkbox"/>
A <u>fractional</u> power indicates a <u>root</u> a power of $\frac{1}{2}$ means 'square root'. $25^{\frac{1}{2}} = \sqrt{25} = 5$ a power of $\frac{1}{3}$ means 'cube root'. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$	(4) $4^{\frac{5}{2}}$	<input type="checkbox"/>
	(5) $32^{\frac{3}{5}}$	<input type="checkbox"/>

<p><u>Example:</u> $144^{-\frac{1}{2}} = \frac{1}{144^{\frac{1}{2}}} = \frac{1}{\sqrt{144}} = \frac{1}{12}$ <input type="checkbox"/></p> <p>Tick the box when you understand.</p>	<p>(6) $16^{-\frac{7}{4}}$</p>	<input type="checkbox"/>
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Indices continued (What you need for AS level)

It is very useful to mathematicians to be able to write algebraic expressions in different ways and one of the most important ways is in the form (number) x^{power}



Examples of writing things in the form αx^n . Tick the box when you understand.	Now try Exercise 3: Write these in the form αx^n . Tick when correct.
$\frac{2x}{3} = \left(\frac{2}{3}\right)\left(\frac{x}{1}\right)$ $= \frac{2}{3}x$ <input type="checkbox"/>	<p>(1) $\frac{x}{5} =$</p> <input type="checkbox"/>
$\frac{2}{5x} = \left(\frac{2}{5}\right)\left(\frac{1}{x}\right)$ $= \frac{2}{5}x^{-1}$ <input type="checkbox"/>	<p>(2) $\frac{3}{2\sqrt{x}} =$</p> <input type="checkbox"/>
$\frac{x}{3\sqrt{x}} = \left(\frac{1}{3}\right)\left(\frac{x}{\sqrt{x}}\right)$ $= \frac{1}{3}x^{1-\frac{1}{2}}$ $= \frac{1}{3}x^{\frac{1}{2}}$ <input type="checkbox"/>	<p>(3) $\frac{\sqrt{x}}{3x^2} =$</p> <input type="checkbox"/>
$2\sqrt{16x^3} = 2\sqrt{16}\sqrt{x^3}$ $= 8(x^3)^{\frac{1}{2}}$ $= 8x^{\frac{3}{2}}$ <input type="checkbox"/>	<p>(4) $\sqrt[3]{8x^2} =$</p> <input type="checkbox"/>

$$\begin{aligned}\frac{2+x}{\sqrt{x}} &= \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{x}} \\ &= \left(\frac{2}{1}\right)\left(\frac{1}{x^{\frac{1}{2}}}\right) + x^{\left(1-\frac{1}{2}\right)} \\ &= 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}\end{aligned}$$

*Note: This one
has two terms so
is written in the
form*

$$(5) \quad \frac{2\sqrt{x} + 4}{x^2} =$$

More practice of **the most important type of indices...** Write these in the form $\alpha x^n + \beta x^m$. Tick the boxes when they are correct.

(6)
$$\frac{2x-4}{3x^2} = \frac{2x}{3x^2} - \frac{4}{3x^2}$$

$$= \left(\frac{2}{3}\right)\left(\frac{x}{x^2}\right) - \left(\frac{4}{3}\right)\left(\frac{1}{x^2}\right)$$

$$= \frac{2}{3}x^{-1} - \frac{4}{3}x^{-2}$$

☐

(7)
$$\frac{1-4x}{4x^3} =$$

☐

(8)
$$\frac{1-4\sqrt{x}}{x} =$$

☐

Understand?.....

(9)
$$\frac{x^2-3}{\sqrt{x}} =$$

☐

(10)
$$\frac{x-2}{x^2} =$$

☐

(11)
$$\frac{2+\sqrt{x}}{\sqrt{x}} =$$

☐

(12)
$$\frac{2x+4}{4x} =$$

☐

(13)
$$\frac{\sqrt{x}+6}{3x^2} =$$

☐

(14)
$$\frac{2x-1}{x^2} =$$

☐

we had started by cubing both sides rather than square rooting. It would still work but it would have been more difficult.	With this question, is it easiest to start by cube rooting each side or by raising each side to the power 4?
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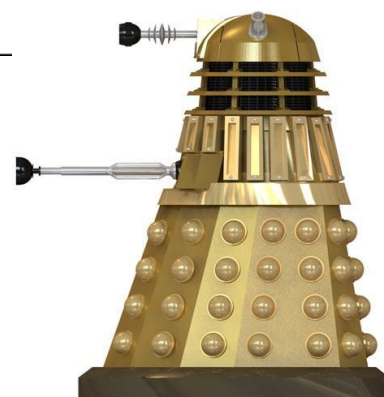
Part A – Section 3 - SURDS

A surd is an IRRATIONAL ROOT e.g., $\sqrt{2}$, $\sqrt{3}$, etc., but not $\sqrt[3]{8}$ because $\sqrt[3]{8} = 2$

ANOTHER TOP TIP! When you write a root, make sure that it has a top which goes over everything in the root otherwise things can jump out without you noticing. $\sqrt{4x}$ could mean which is $2x$ or it might mean which is .

NO!!

WRONG!!!
Students often make up the rule that a power can be applied to the two terms of a sum separately. Actually, nothing can be done to simplify this expression.



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Examples. Tick when you understand.	Now try exercise 4: Simplify into the form $a\sqrt{b}$. Tick when correct.	
<u>Multiplication and roots:</u> $\sqrt{ab} = \sqrt{a}\sqrt{b}$ $\sqrt{80} = \sqrt{16}\sqrt{5}$ $= 4\sqrt{5}$	(1) $\sqrt{27} =$	<input type="checkbox"/>
	(2) $\sqrt{45} =$	<input type="checkbox"/>
	(3) $\sqrt{12} =$	<input type="checkbox"/>
	(4) $\sqrt{48} =$	<input type="checkbox"/>
	(5) $\sqrt{75} =$	<input type="checkbox"/>
<u>Division and roots:</u> $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	(6) $\frac{\sqrt{12}}{2} =$	<input type="checkbox"/>

$\sqrt{5\frac{4}{9}} = \sqrt{\frac{49}{9}}$ $= \frac{\sqrt{49}}{\sqrt{9}}$ $= \frac{7}{3}$ <div style="border: 1px solid black; width: 40px; height: 20px; margin: 10px auto;"></div> <div style="border: 2px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> <p>Can you see that top heavy fractions are much nicer than mixed numbers or decimals?!</p> </div>	$(7) \frac{\sqrt{98}}{7} =$ <div style="border: 1px solid black; width: 40px; height: 20px; float: right;"></div>
	$(8) \frac{\sqrt{18}}{\sqrt{2}} =$ <div style="border: 1px solid black; width: 40px; height: 20px; float: right;"></div>
	$(9) \frac{\sqrt{27}}{\sqrt{3}} =$ <div style="border: 1px solid black; width: 40px; height: 20px; float: right;"></div>

Simplifying surds

Example Simplifying and collecting like terms. Tick the box when you understand.	
$\begin{aligned} \sqrt{75} + 2\sqrt{48} - 5\sqrt{12} &= \sqrt{(25)(3)} + 2\sqrt{(16)(3)} - 5\sqrt{(4)(3)} \\ &= \sqrt{25}\sqrt{3} + 2\sqrt{16}\sqrt{3} - 5\sqrt{4}\sqrt{3} \\ &= 5\sqrt{3} + 2(4)\sqrt{3} - 5(2)\sqrt{3} \\ &= 5\sqrt{3} + 8\sqrt{3} - 10\sqrt{3} \\ &= 3\sqrt{3} \end{aligned}$ <div style="border: 1px solid black; width: 40px; height: 20px; float: right;"></div>	
Exercise 4 continued. Tick when correct.	
(10) $\sqrt{12} + 3\sqrt{75} =$	<div style="border: 1px solid black; width: 40px; height: 20px; float: right;"></div>
(11) $\sqrt{200} + \sqrt{18} - 2\sqrt{72} =$	

<div style="text-align: right; border: 1px solid black; width: 50px; height: 20px; margin: 0 auto;"></div>
<p>(12) $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80} =$</p> <div style="text-align: right; border: 1px solid black; width: 50px; height: 20px; margin: 0 auto;"></div>

RATIONALISING THE DENOMINATOR

This means write the fraction differently, so there is no surd on the bottom.

<p>TYPE 1 Examples: Multiplying the top and bottom by the surd on the bottom. Tick when understood.</p>	<p>Exercise 5: Rationalise the denominators and write in the form $a\sqrt{b}$ (where a is usually a fraction). Tick when correct.</p>
$\frac{1}{\sqrt{3}}$ $= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $= \frac{\sqrt{3}}{3}$ $= \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{1}\right)$ $= \frac{1}{3}\sqrt{3}$ <div style="text-align: right; border: 1px solid black; width: 50px; height: 20px; margin: 0 auto;"></div>	<div style="border-bottom: 1px solid black; padding-bottom: 10px;"> <p>(1) $\frac{1}{\sqrt{2}} =$</p> <div style="text-align: right; border: 1px solid black; width: 50px; height: 20px; margin: 0 auto;"></div> </div> <div style="padding-top: 10px;"> <p>(2) $\frac{1}{\sqrt{7}} =$</p> <div style="text-align: right; border: 1px solid black; width: 50px; height: 20px; margin: 0 auto;"></div> </div>

$\frac{1}{4\sqrt{2}}$ $= \frac{\sqrt{2}}{4\sqrt{2}\sqrt{2}}$ $= \frac{\sqrt{2}}{4(2)}$ $= \left(\frac{1}{8}\right)\left(\frac{\sqrt{2}}{1}\right)$ $= \frac{1}{8}\sqrt{2}$ <div style="text-align: right;"><input type="text"/></div>	<div> $(3) \frac{7}{\sqrt{5}} =$ <div style="text-align: right;"><input type="text"/></div> </div> <div> $(4) \frac{\sqrt{2}}{3\sqrt{3}} =$ <div style="text-align: right;"><input type="text"/></div> </div>
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If the denominator is a sum or difference you can use the clever technique of multiplying top and bottom by the ‘opposite’ of the denominator to create a difference of two squares on the bottom:

$$(a - b)(a + b) = a^2 - b^2$$

<p>TYPE 2 Examples. Multiply top and bottom by the ‘opposite’ of the bottom. Follow the example carefully then try to do it yourself. Tick when understood.</p>	<p>Now try Exercise 6: Rationalise the denominators and write in the form $a + b\sqrt{c}$. Tick when correct.</p>
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The bottom is $1+\sqrt{3}$ so we multiply top and bottom by $1-\sqrt{3}$

$$\frac{1}{1+\sqrt{3}} = \frac{(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

$$= \frac{1-\sqrt{3}}{(1)^2 - (\sqrt{3})^2}$$

$$= \frac{1-\sqrt{3}}{1-3}$$

$$= \frac{1-\sqrt{3}}{-2}$$



Do you recognise this step from exercise 3?

$$= \frac{1}{-2} - \frac{\sqrt{3}}{-2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= -\frac{1}{2} + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{1}\right)$$

$$= -\frac{1}{2} + \frac{1}{2}\sqrt{3} \quad \boxed{}$$

(1) $\frac{1}{1+\sqrt{2}} =$

The bottom is $4-\sqrt{2}$ so we multiply top and bottom by $4+\sqrt{2}$

$$\frac{3}{4-\sqrt{2}} = \frac{3(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})}$$

$$= \frac{3(4+\sqrt{2})}{(4)^2 - (\sqrt{2})^2}$$

$$= \frac{3(4+\sqrt{2})}{16-2}$$

$$= \frac{12+3\sqrt{2}}{14}$$



$$= \frac{12}{14} + \frac{3\sqrt{2}}{14}$$

$$= \frac{6}{7} + \left(\frac{3}{14}\right)\left(\frac{\sqrt{2}}{1}\right)$$

$$= \frac{6}{7} + \frac{3}{14}\sqrt{2} \quad \boxed{}$$

Important step!

(2) $\frac{5}{1-\sqrt{3}} =$

Part A Mini-Test

So, you've completed all the exercise in part A. Well done! Did you remember to copy the common mistakes you found into the table at the front of the booklet? This is really important!

The important question now is whether your brain has really learned the techniques in part A. To find out, use this mini-test in exam conditions then mark it yourself using the answers at the back of the booklet and give yourself a score. You should aim for 25/25 of course but certainly anything less than 15/25 should be a worry. Each question number comes from that number exercise. **Go back to the exercises containing the questions you got wrong** then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes. No Calculator allowed.
Good Luck!

- 1 (a) Write $\frac{3x}{4} \times 5$ as a single fraction
(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction
- 2 (a) Evaluate $32^{\frac{3}{5}}$
(b) Evaluate $9^{-\frac{1}{2}}$
- 3 (a) Write $\frac{3}{2\sqrt{x}}$ in the form αx^n
(b) Write $\frac{2\sqrt{x} + 4}{x^2}$ in the form $\alpha x^n + \beta x^m$
(c) Solve the equation $x^{-\frac{2}{3}} = 9$
- 4 (a) Simplify $\sqrt{45}$
(b) Simplify $\frac{\sqrt{12}}{2}$
(c) Simplify $\sqrt{200} + \sqrt{18} - 2\sqrt{72}$
- 5 Rationalise the denominator of $\frac{7}{\sqrt{5}}$ leaving your answer in the form $a\sqrt{5}$
- 6 Rationalise the denominator of $\frac{1}{1 + \sqrt{2}}$

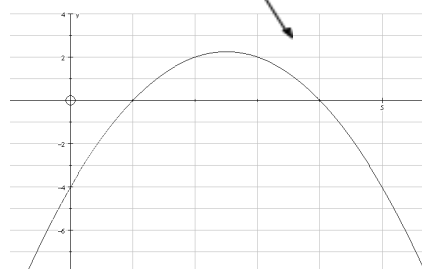
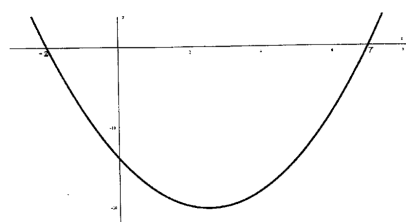
**Mark your test using the solutions at the back of the booklet
and put your score here /25**

PART B - QUADRATICS

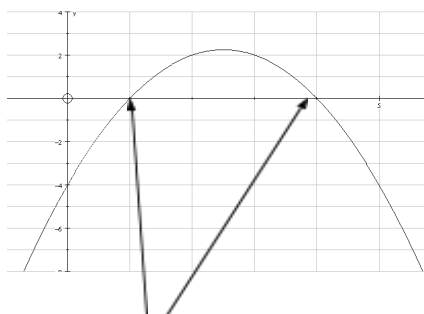
You should know what a 'quadratic' is but in order to start AS you need to REALLY understand and be able to use quadratics. You need to be able to manipulate quadratic expressions by factorising and completing the square and you need to be able to solve quadratic equations using 3 different methods.

A **QUADRATIC EXPRESSION** is just some algebra written in the form $ax^2 + bx + c$. The numbers a , b and c can be anything you like (b and c could even be zero!). It is usually given the name y or $f(x)$.

A **QUADRATIC GRAPH** looks like this depending on whether a is positive or negative:



A **QUADRATIC EQUATION** can always be rearranged to make the right hand side equal to zero, i.e., so that it is in the form $ax^2 + bx + c = 0$. The solutions can be seen (where the graph crosses the x -axis). Normally, you would expect there to be two possible answers, as in the graphs above.



Solutions to the equation $ax^2 + bx + c = 0$

Of course, if the quadratic graph is totally above or below the x axis then it will never cross the x axis. In these cases, the quadratic equation has no solutions. Or possibly the quadratic graph might just sit on the x axis rather than crossing it, in which case the quadratic equation will only have one solution (called a repeated root).

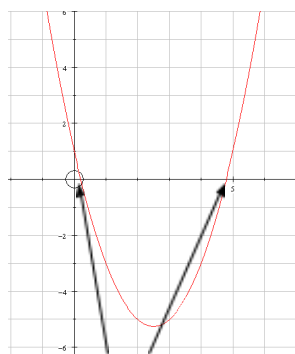
Can we solve the equation $2x^2 + 6x = 8$?!

First get everything on the left hand side so it equals zero..... $2x^2 + 6x - 8 = 0$.

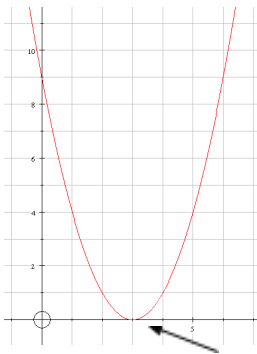
You are now ready to solve the equation – if it can be solved..... This quadratic might have 2 solutions like in the picture above, it might have one solution or it might have no solutions. Over the next few pages, you will first practise working out whether it has none, one or two solutions. Then you will practise finding the solutions (if they exist!) by three different methods.

Part B – Section 1 – THE DISCRIMINANT

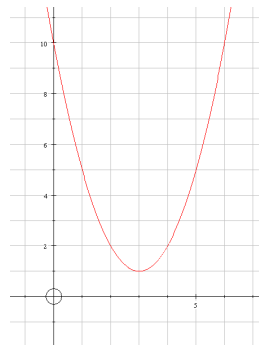
All quadratic graphs cross the y -axis. The y -intercept is the value of the quadratic when $x = 0$. The behaviour on the x -axis is a bit more complicated. Some quadratic graphs cross the x -axis twice, giving two solutions to the equation $ax^2 + bx + c = 0$. Other quadratics simply 'sit' on the x axis, so they only have one solution to the equation $ax^2 + bx + c = 0$. There are also some quadratics which don't cross the x axis at all so these quadratics have no solutions to the equation $ax^2 + bx + c = 0$.



Two distinct roots



One repeated root
(two equal roots)



No real roots

The solutions of an equation, i.e., the places where the graph crosses the x -axis, are called the **roots of the equation**.

We know that the solutions to a quadratic equation are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What could go wrong? Why do we sometimes get two solutions, sometimes one solution and sometimes no solutions?! The answer lies inside the square root sign.

$$b^2 - 4ac > 0 \text{ (positive)}$$

Everything is fine. We square root $b^2 - 4ac$ and get two solutions using the quadratic formula.

$$b^2 - 4ac = 0$$

If $b^2 - 4ac = 0$ then $\sqrt{b^2 - 4ac} = 0$ so in this case $x = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$. Just one (repeated) solution.

$$b^2 - 4ac < 0 \text{ (negative)}$$

If $b^2 - 4ac < 0$ have a problem. We can't square root a negative number so we are stuck. That is why, in this situation, there are no solutions.

$b^2 - 4ac$ is called the DISCRIMINANT of the quadratic because it helps us to discriminate between the quadratics with no roots, quadratics with one repeated root and quadratics with two roots.

Sometimes the symbol Δ is used to refer to the discriminant of a quadratic. Now go back to page 4 of this booklet and look at the quadratics at the bottom of the page. Does it make sense to you that you can see whether the discriminant is positive, negative or zero by looking at the graph of the quadratic?

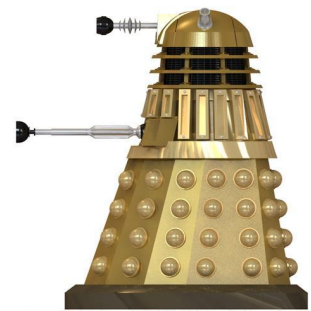
It is important to remember that in the discriminant, $b^2 - 4ac$, a represents the amount of x^2 in your quadratic, b represents the amount of x in your quadratic and c represents everything else in your quadratic (ie the numbers). **Don't let yourself get muddled if the quadratic is written in a funny order!**

Exercise 7 Write down the discriminant of each of these quadratics and hence state whether each one has two roots, one repeated root or no roots. Tick when correct.

Note: make sure that you square all of b !

If b is -6 then b^2 is $(-6)^2 = 36$ NOT -36

If b is $2k$ then $b^2 = (2k)^2 = 4k^2$ NOT $2k^2$



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Quadratic	Value of Discriminant	Circle the number of roots
EXAMPLE (1) $x^2 + 8x + 7$	$(8)^2 - 4(1)(7) = 36$ $36 > 0$	None One Repeated <u>Two</u> <input type="checkbox"/>
(2) $3x + x^2 - 2$	$()^2 - 4()() =$	None One Repeated Two <input type="checkbox"/>
(3) $x^2 + 3$	$()^2 - 4()() =$	None One Repeated Two <input type="checkbox"/>
(4) $2x^2 + 3 - 6x$	$()^2 - 4()() =$	None One Repeated Two <input type="checkbox"/>
(5) $x - x^2$	$()^2 - 4()() =$	None One Repeated Two <input type="checkbox"/>
(6) $x^2 - 6x + 9$	$()^2 - 4()() =$	None One Repeated Two <input type="checkbox"/>

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Part B – Section 2- FACTORISING QUADRATICS

... Using the difference of two squares $(a)^2 - (b)^2 = (a - b)(a + b)$

	Exercise 8 Factorise the following	Tick when correct
<u>Example 1</u> $x^2 - 9 = (x - 3)(x + 3)$	(1) $x^2 - 1$	<input type="checkbox"/>
	(2) $4x^2 - 9$	<input type="checkbox"/>
<u>Example 2</u> $9x^2 - 16 = (3x)^2 - (4)^2$ $= (3x - 4)(3x + 4)$	(3) $49 - x^2$	<input type="checkbox"/>
	(4) $2x^2 - 8$	<input type="checkbox"/>
	(5) $x^2 - 16$	<input type="checkbox"/>
<u>Example 3</u> $8x^2 - 2 = 2(4x^2 - 1)$ $= 2((2x)^2 - 1^2)$ $= 2(2x - 1)(2x + 1)$	(6) $9x^2 - 1$	<input type="checkbox"/>
	(7) $36 - 25x^2$	<input type="checkbox"/>
	(8) $9x^2 - 36$	<input type="checkbox"/>

Exercise 9 Factorise the following quadratics. Remember to expand out to check your answers. The first one has been completed for you. Tick when correct!

(1) $x^2 - 2x - 15$	(2) $6x^2 - 3x$	(3) $x^2 - 5x - 6$
$= (x - 3)(x + 5)$ Check: $(x - 3)(x + 5)$ $= x^2 - 3x + 5x - 15$ $= x^2 - 2x - 15$		
<input type="text"/>	<input type="text"/>	<input type="text"/>
(4) $x^2 + x - 6$	(5) $2x^2 + 6x$	(6) $x^2 - 6x - 16$
<input type="text"/>	<input type="text"/>	<input type="text"/>

Part B – Section 3 - COMPLETING THE SQUARE

Completing the square is a bit like factorising. It doesn't change the quadratic but it changes the way the quadratic expression is written.

When we factorise, we change $x^2 + bx + c$ into $(x - p)(x - q)$ by finding p and q

When we complete the square, we change $x^2 + bx + c$ into $(x + B)^2 + C$ by finding B and C

$$x^2 + bx + c = (x + \text{half of } b)^2 - (\text{half of } b)^2 + c$$

	Exercise 11 Complete the square of the following quadratics.	
Example Express $x^2 + 6x + 11$ in the completed square form $(x + B)^2 + C$. $x^2 + 6x + 11 = (x + 3)^2 - (3)^2 + 11$ $= (x + 3)^2 - 9 + 11$ $= (x + 3)^2 + 2$ <input type="text"/>	(1) $x^2 + 8x + 7$	(2) $x^2 - 2x - 15$
	<input type="text"/>	<input type="text"/>
Example 2 Express $x^2 - 10x + 13$ in the completed square form $(x + B)^2 + C$. $x^2 - 10x + 13 = (x - 5)^2 - (-5)^2 + 13$ $= (x - 5)^2 - 25 + 13$ $= (x - 5)^2 - 12$ <input type="text"/>	(3) $x^2 + 6x + 10$	(4) $x^2 - 10x + 9$
	<input type="text"/>	<input type="text"/>
(5) $x^2 + 12x + 100$	(6) $x^2 + 2x - 6$	(7) $x^2 + 6x - 5$
<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

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Part B – Section 4 - SOLVING QUADRATICS

There are 3 ways to solve a quadratic equation:- by factorising, by using the quadratic formula or by completing the square.

- Factorising uses the fact that if 2 things multiply together to make zero then one of them MUST be zero. You can't always factorise a quadratic even if it has solutions.
- The quadratic formula will always give you the solutions, so long as there are some!
- Completing the square allows you to simply rearrange the quadratic to find x . If there are solutions to the quadratic equation then completing the square will always work.

Example - Factorising

$$2x^2 - 5x + 3 = 0$$

Factorising gives: $(2x - 3)(x - 1) = 0$

so either $2x - 3 = 0$ or $x - 1 = 0$

$$2x = 3$$

$$\therefore x = \frac{3}{2} \quad \text{or} \quad x = 1$$

This means that the graph of the quadratic function $f(x) = 2x^2 - 5x + 3$ crosses the x axis at $\frac{3}{2}$ and 1.

Tick when understood ☐

Factorising

Exercise 12 Solve the following quadratic equations by factorising. Tick when correct.

(1) $x^2 + 11x + 28 = 0$	(2) $x^2 + 3x = 0$	(3) $2x^2 + 3x - 14 = 0$
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Now try Exercise 13: Solve the following quadratic equations using the quadratic formula and leave your answers in the form

$$x = A \pm B\sqrt{C} \text{ as}$$

in the example on the left.

To solve the quadratic equation $ax^2 + bx + c = 0$ you can use the quadratic formula above.

Then you will need to rearrange these answers into the form

$$x = A \pm B\sqrt{C}$$

Example – Using the formula

Solve $x^2 + 3x + 1 = 0$

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2 \times 1} \\ &= \frac{-3 \pm \sqrt{9 - 4}}{2} \\ &= \frac{-3 \pm \sqrt{5}}{2} \\ &= -\frac{3}{2} \pm \frac{\sqrt{5}}{2} \\ &= -\frac{3}{2} \pm \left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right) \\ &= -\frac{3}{2} \pm \frac{1}{2}\sqrt{5} \end{aligned}$$

Important step!

Tick when understood

☐

(1) $2x^2 + 4x + 1 = 0$

(2) $x^2 - 7x + 9 = 0$

nicely in the form you want: $x = A \pm B\sqrt{C}$. In the examples below we have shown every tiny step to help you to follow what is happening.

Example.	Now try Exercise 14 Solve this quadratic by completing the square. Tick when correct.
<p>Solve $x^2 + 3x + 1 = 0$ by completing the square</p> $\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 1 = 0$ $\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{4}{4} = 0$ $\left(x + \frac{3}{2}\right)^2 - \frac{5}{4} = 0$ $\left(x + \frac{3}{2}\right)^2 = \frac{5}{4}$ $x + \frac{3}{2} = \pm \sqrt{\frac{5}{4}}$ $x + \frac{3}{2} = \pm \frac{\sqrt{5}}{\sqrt{4}}$ $x + \frac{3}{2} = \pm \left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right)$ $x = -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$ $x = -\frac{3}{2} + \frac{1}{2}\sqrt{5} \text{ or } x = -\frac{3}{2} - \frac{1}{2}\sqrt{5}$	<p>(1) $x^2 + 2x - 6 = 0$</p>

Example – Using the Completed Square to solve a quadratic.	Exercise 14 continued Solve this quadratic by completing the square. Tick when correct.
<p>Solve $x^2 - x = 0$ by completing the square</p> <div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="width: 45%;"> $\left(x - \frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 = 0$ $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = 0$ $\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$ $x - \frac{1}{2} = \pm \sqrt{\frac{1}{4}}$ $x - \frac{1}{2} = \pm \frac{\sqrt{1}}{\sqrt{4}}$ $x - \frac{1}{2} = \pm \frac{1}{2}$ $x = \frac{1}{2} - \frac{1}{2} \text{ or } x = \frac{1}{2} + \frac{1}{2}$ $x = 0 \quad \text{or} \quad x = 1$ </div> <div style="width: 50%;"> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; margin-bottom: 10px;"> <p>First complete the square, then expand out the (half b)² bit. In this question, $c = 0$</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; margin-bottom: 10px;"> <p>Put the number on the right hand side then square root both sides, remembering to add the \pm sign!</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; margin-bottom: 10px;"> <p>Remember that the square root of a fraction is the square root of the top over the square root of the bottom.</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 10px;"> <p>Finally move the 'half of b' to the other side so it $= x$ says</p> </div> </div> </div> <div style="margin-top: 20px;"> <p>Tick when understood <input type="checkbox"/></p> </div>	<p>(2) $x^2 + 6x - 5 = 0$</p> <div style="text-align: right; margin-top: 100px;"> <input type="checkbox"/> </div>

Part B Mini-Test

So, you've completed all the exercise in part B. Well done! The important question is whether your brain has really learned these techniques. To find out, use this mini test in exam conditions then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes. No Calculator allowed.

Good Luck!

- 7 Evaluate the discriminant of the quadratic $y = 2x^2 + 3 - 6x$ and hence state the number of roots of the equation $2x^2 + 3 - 6x = 0$
- 8 Factorise the quadratic $y = 4x^2 - 9$ using the difference of two squares.
- 9 Factorise the quadratic $y = 2x^2 + 6x$
- 10 Factorise the quadratic $y = 3x^2 - 13x - 10$
- 11 Write the quadratic $y = x^2 + 8x + 7$ in completed square form.
- 12 Solve the equation $x^2 + 3x = 0$ by factorising.
- 13 Solve the equation $2x^2 + 4x + 1 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.
- 14 Solve the equation $x^2 + 6x - 5 = 0$ by rearranging the completed square, leaving the answer(s) in surd form.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Completed square:

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

**Mark your test using the solutions at the back of the booklet
and put your score here /20**

ARE YOU READY FOR AS?

In order to be confident starting AS maths you need to be confident with the techniques in this booklet. When you start the course we will give you a test like this one to check that you are ready to start AS. Try this test in exam conditions then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 1 hour. No Calculator allowed.
Good Luck!

1(a) Write $\frac{3x}{2} \div 5$ as a single fraction

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction

2(a) Evaluate $16^{-\frac{7}{4}}$

(b) Evaluate $4^{\frac{5}{2}}$

3(a) Write $\frac{2 + \sqrt{x}}{\sqrt{x}}$ in the form $\alpha x^n + \beta x^m$

(b) Solve the equation $x^{\frac{3}{4}} = \frac{1}{27}$

4(a) Simplify $\sqrt{48}$

(b) Simplify $\frac{\sqrt{18}}{\sqrt{2}}$

(c) Simplify $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}$

5(a) Rationalise the denominator of $\frac{\sqrt{2}}{3\sqrt{3}}$ leaving your answer in the form $a\sqrt{6}$

(b) Rationalise the denominator of $\frac{1}{\sqrt{2}}$

Quadratic formula:
Completed square:

6 Ra
 $\frac{5}{1 - \sqrt{3}}$

Staple your completed test into your booklet so that you can discuss with your teacher in September.

- 7 Evaluate the discriminant of the quadratic $y = x - x^2$ and hence state the number of roots of the equation $x - x^2 = 0$
- 8 Factorise the quadratic $y = 2x^2 - 8$ using the difference of two squares
- 9 Factorise the quadratic $y = 6x^2 - 3x$
- 10 Factorise the quadratic $y = 2x^2 - 11x + 12$
- 11 Write the quadratic $y = x^2 - 6x - 16$ in completed square form.
- 12 Solve the equation $2x^2 + 3x - 14 = 0$ by factorising.
- 13 Solve the equation $x^2 - 7x + 9 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.
- 14 Solve the equation $x^2 + 2x - 6 = 0$ by rearranging the completed square, leaving the answer(s) in surd form.

**Mark your test using the
solutions at the back of
the booklet and put your
score here /40**

ANSWERS

Exercise 1

(1) $\frac{15x}{4}$	(2) $\frac{2x+3}{x^2}$	(3) $\frac{3x}{10}$
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Exercise 2

(1) $\frac{1}{64}$	(2) $\frac{1}{3}$	(3) $\frac{1}{3}$	(4) 32	(5) 8	(6) $\frac{1}{128}$
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Exercise 3

(1) $\frac{1}{5}x$	(2) $\frac{3}{2}x^{-\frac{1}{2}}$	(3) $\frac{1}{3}x^{\frac{3}{2}}$	(4) $2x^{\frac{2}{3}}$
(5) $2x^{-\frac{3}{2}} + 4x^{-2}$	(6) $\frac{2}{3}x^{-1} + \frac{4}{3}x^{-2}$	(7) $\frac{1}{4}x^{-3} - x^{-2}$	(8) $x^{-1} - 4x^{\frac{1}{2}}$
(9) $x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$	(10) $x^{-1} - 2x^{-2}$	(11) $2x^{-\frac{1}{2}} + 1$	(12) $\frac{1}{2} + x^{-1}$
(13) $\frac{1}{3}x^{-\frac{3}{2}} + 2x^{-2}$	(14) $2x^{-1} - x^{-2}$		
(15) $x = \pm \frac{1}{27}$	(16) $x = \pm 32$	(17) $x = \frac{1}{81}$	

Exercise 4

(1) $3\sqrt{3}$	(2) $3\sqrt{5}$	(3) $2\sqrt{3}$	(4) $4\sqrt{3}$	(5) $5\sqrt{3}$	(6) $\sqrt{3}$
(7) $\sqrt{2}$	(8) 3	(9) 3	(10) $17\sqrt{3}$	(11) $\sqrt{2}$	(12) $-4\sqrt{5}$

Exercise 5

(1) $\frac{1}{2}\sqrt{2}$	(2) $\frac{1}{7}\sqrt{7}$	(3) $\frac{7}{5}\sqrt{5}$	(4) $\frac{1}{9}\sqrt{6}$
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Exercise 6

(1) $-1 + \sqrt{2}$	(2) $-\frac{5}{2} - \frac{5}{2}\sqrt{3}$
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Exercise 7

(1) 36, two	(2) 17, two	(3) - 12, none
(4) 12, two	(5) 1, two	(6) 0, one repeated

Exercise 8

(1) $(x-1)(x+1)$	(2) $(2x-3)(2x+3)$	(3) $(7-x)(7+x)$
(4) $2(x-2)(x+2)$	(5) $(x-4)(x+4)$	(6) $(3x-1)(3x+1)$
(7) $(6-5x)(6+5x)$	(8) $9(x-2)(x+2)$	

Exercise 9

(1) $(x+3)(x-5)$	(2) $3x(2x-1)$	(3) $(x-6)(x+1)$
(4) $(x-2)(x+3)$	(5) $2x(x+3)$	(6) $(x-8)(x+2)$

Exercise 10

(1) $(2x+1)(x+2)$	(2) $(3x-2)(x-2)$	(3) $(2x+3)(x+2)$
(4) $(3x+2)(x-5)$	(5) $(2x-1)(x+5)$	(6) $(2x-3)(x-4)$

Exercise 11

(1) $(x+4)^2 - 9$	(2) $(x-1)^2 - 16$	(3) $(x+3)^2 + 1$
(4) $(x-5)^2 - 16$	(5) $(x+6)^2 + 64$	(6) $(x+1)^2 - 7$
(7) $(x+3)^2 - 14$		

Exercise 12

(1) $x = -7$ or -4	(2) $x = 0$ or -3	(3) $x = -7/2$ or 2
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Exercise 13

$$(1) \quad x = -1 \pm \frac{1}{2}\sqrt{2} \qquad (2) \quad x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$$

Exercise 14

(1) $x = -1 + \sqrt{7}$ or $x = -1 - \sqrt{7}$	(2) $x = -3 + \sqrt{14}$ or $x = -3 - \sqrt{14}$
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Part A Mini Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!) , 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Give yourself a bonus mark if you got (6b) correct ☺. The total test is out of 25 and **anything below 15/25 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

$$1 \quad (a) \quad \frac{3x}{4} \times 5 = \left(\frac{3x}{4}\right)\left(\frac{5}{1}\right) = \frac{15x}{4} \qquad 2 \quad (a) \quad 32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$$

$$1 \quad (b) \quad \frac{2}{x} + \frac{3}{x^2} = \frac{2x}{x^2} + \frac{3}{x^2} = \frac{2x+3}{x^2} \qquad 2 \quad (b) \quad 9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$3 \quad (a) \quad \frac{3}{2\sqrt{x}} = \left(\frac{3}{2}\right)\left(\frac{1}{\sqrt{x}}\right) = \frac{3}{2}x^{-\frac{1}{2}}$$

$$(b) \quad \frac{2\sqrt{x}+4}{x^2} = \frac{2\sqrt{x}}{x^2} + \frac{4}{x^2} = \left(\frac{2}{1}\right)\left(\frac{\sqrt{x}}{x^2}\right) + \left(\frac{4}{1}\right)\left(\frac{1}{x^2}\right) = 2x^{\frac{1}{2}-2} + 4x^{-2} = 2x^{-\frac{3}{2}} + 4x^{-2}$$

(c)

$$x^{-\frac{2}{3}} = 9$$

$$x^{\frac{2}{3}} = \frac{1}{9}$$

$$x^{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{9}}$$

$$x^{\frac{1}{3}} = \pm \frac{1}{3}$$

$$x = \left(\pm \frac{1}{3}\right)^3$$

$$x = \pm \frac{1}{27}$$

$$4 \quad (a) \quad \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$$

$$(b) \quad \frac{\sqrt{12}}{2} = \frac{\sqrt{4}\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$(c) \quad \sqrt{200} + \sqrt{18} - 2\sqrt{72} = \sqrt{100}\sqrt{2} + \sqrt{9}\sqrt{2} - 2\sqrt{36}\sqrt{2} = 10\sqrt{2} + 3\sqrt{2} - 12\sqrt{2} = \sqrt{2}$$

$$5 \quad \frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}}{5} = \left(\frac{7}{5}\right)\left(\frac{\sqrt{5}}{1}\right) = \frac{7}{5}\sqrt{5}$$

$$6 \quad \frac{1}{1+\sqrt{2}} = \frac{1(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = \frac{1}{-1} - \frac{\sqrt{2}}{-1} = -1 + \sqrt{2}$$

Part B Mini Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!), 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Give yourself 2 bonus marks if you got (7) correct and 2 bonus marks if you got (8) correct ☺.

The total test is out of 20 and **anything below 12/20 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

$$7 \quad b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12. \quad 12 > 0 \text{ hence the equation has 2 distinct roots.}$$

$$8 \quad 4x^2 - 9 = (2x - 3)(2x + 3)$$

$$9 \quad 2x^2 + 6x = 2x(x + 3)$$

$$10 \quad 3x^2 - 13x - 10 = (3x + 2)(x - 5)$$

$$11 \quad x^2 + 8x + 7 = (x + 4)^2 - (4)^2 + 7 = (x + 4)^2 - 9$$

12

$$x^2 + 3x = 0$$

$$x(x + 3) = 0$$

$$x = 0, \quad x + 3 = 0$$

$$x = 0, \quad x = -3$$

13

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{16 - 8}}{4} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4}{4} \pm \frac{\sqrt{8}}{4}$$

$$= -1 \pm \frac{\sqrt{4}\sqrt{2}}{4} = -1 \pm \left(\frac{2}{4}\right)\left(\frac{\sqrt{2}}{1}\right) = -1 \pm \frac{1}{2}\sqrt{2}$$

14

$$x^2 + 6x - 5 = 0$$

$$(x+3)^2 - (3)^2 - 5 = 0$$

$$(x+3)^2 - 14 = 0$$

$$(x+3)^2 = 14$$

$$x+3 = \pm\sqrt{14}$$

$$x = -3 \pm \sqrt{14}$$

Are you ready for AS? Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!), 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! The total test is out of 40 and **anything below 24/40 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

$$1a) \frac{3x}{2} \div 5 = \frac{3x}{2} \times \frac{1}{5} = \frac{3x}{10}$$

$$b) \frac{2}{x} + \frac{3}{x^2} = \frac{2x}{x^2} + \frac{3}{x^2} = \frac{2x+3}{x^2}$$

$$2a) 16^{-\frac{7}{4}} = \frac{1}{16^{\frac{7}{4}}} = \frac{1}{(16^{\frac{1}{4}})^7} = \frac{1}{2^7} = \frac{1}{128}$$

$$b) 4^{\frac{5}{2}} = (4^{\frac{1}{2}})^5 = (\sqrt{4})^5 = 2^5 = 32$$

$$3a) \frac{2+\sqrt{x}}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} = 2x^{-\frac{1}{2}} + 1$$

$$b) x^{\frac{3}{4}} = \frac{1}{27}$$

$$(x^{\frac{1}{4}})^3 = \frac{1}{27}$$

$$x^{\frac{1}{4}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$$

$$x^{\frac{1}{4}} = \frac{1}{3}$$

$$x = \frac{1^4}{3^4}$$

$$x = \frac{1}{81}$$

$$4a) \sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

$$b) \frac{\sqrt{18}}{\sqrt{2}} = \frac{\sqrt{9 \times 2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$$

c)

$$\sqrt{20} + 2\sqrt{45} - 3\sqrt{80} = \sqrt{4}\sqrt{5} + 2\sqrt{9}\sqrt{5} - 3\sqrt{16}$$

$$5a) \frac{\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2}\sqrt{3}}{3 \times 3} = \frac{1}{9}\sqrt{6}$$

$$b) \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$6) \frac{5}{1-\sqrt{3}} = \frac{5}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{5(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$$

$$= \frac{5+5\sqrt{3}}{1-\sqrt{3}\sqrt{3}}$$

$$= \frac{5+5\sqrt{3}}{-2}$$

$$= -\frac{5}{2} - \frac{5}{2}\sqrt{3}$$

$$7) (1)^2 - 4(-1)(0) = 1, \text{ two roots}$$

$$8) 2x^2 - 8 = 2(x^2 - 4) = 2(x + 2)(x - 2)$$

$$9) 6x^2 - 3x = 3x(2x - 1)$$

$$10) 2x^2 - 11x + 12 = (2x - 3)(x - 4)$$

$$11) x^2 - 6x - 16 = (x - 3)^2 - (-3)^2 - 16 \\ = (x - 3)^2 - 25$$

$$12) 2x^2 + 3x - 14 = 0$$

$$(2x + 7)(x - 2) = 0$$

$$2x + 7 = 0 \text{ or } x - 2 = 0$$

$$x = -\frac{7}{2}, \text{ or } x = 2$$

$$13) x^2 - 7x + 9 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{13}}{2}$$

$$x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$$

$$14) x^2 + 2x - 6 = 0$$

$$(x + 1)^2 - (1)^2 - 6 = 0$$

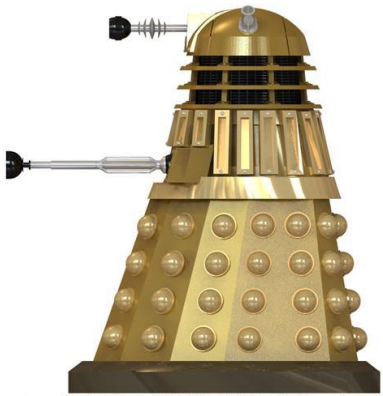
$$(x + 1)^2 = 7$$

$$x + 1 = \pm\sqrt{7}$$

$$x = -1 \pm \sqrt{7}$$

Notes

Use these pages to collect the top 10 most common mistakes made by students (indicated by the Daleks). Writing them down for yourself will help you not avoid doing them yourself.



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Classic Mistake	Correction/Explanation

Classic Mistake	Correction/Explanation

The amount of time this booklet will take to complete will depend entirely on how well you have learned GCSE algebra. You will be best prepared to start AS if you work little and often to allow you to absorb what you are learning, then test yourself on a different day (can you still do the technique a day or a week later?) Getting started early in the holidays will give you a chance to seek help (see page 3), when (not if) you get stuck!

Use the following grid to plan and monitor your progress through the booklet.

	Topic	Exercise	Page	Approx. time required (minutes)	To be completed by	Done	Notes / Test Score
PART A – Learning to Avoid Common Algebraic Mistakes	S1 - Fractions	1	6	10			
	S2 – Indices	2	7	15			
		3	8	20			
	S3 - Surds	4	11	15			
		5	13	10			
		6	14	15			
	PART A Mini-Test		15	30			
PART B – Developing Confidence with Quadratics	S1 – The Discriminant	7	18	15			
	S2 – Factorising Quadratics	8	19	10			
		9	19	10			
		10	20	15			
	S3 – Completing the Square	11	21	20			
	S4 – Solving Quadratics	12	22	10			
		13	23	10			
		14	24	15			
	PART B Mini-Test		26	30			
Are You Ready for AS?			27	60			

If there is anything you would like to ask your teacher, or anything you are worried about, use this space: